J. Michael McCarthy

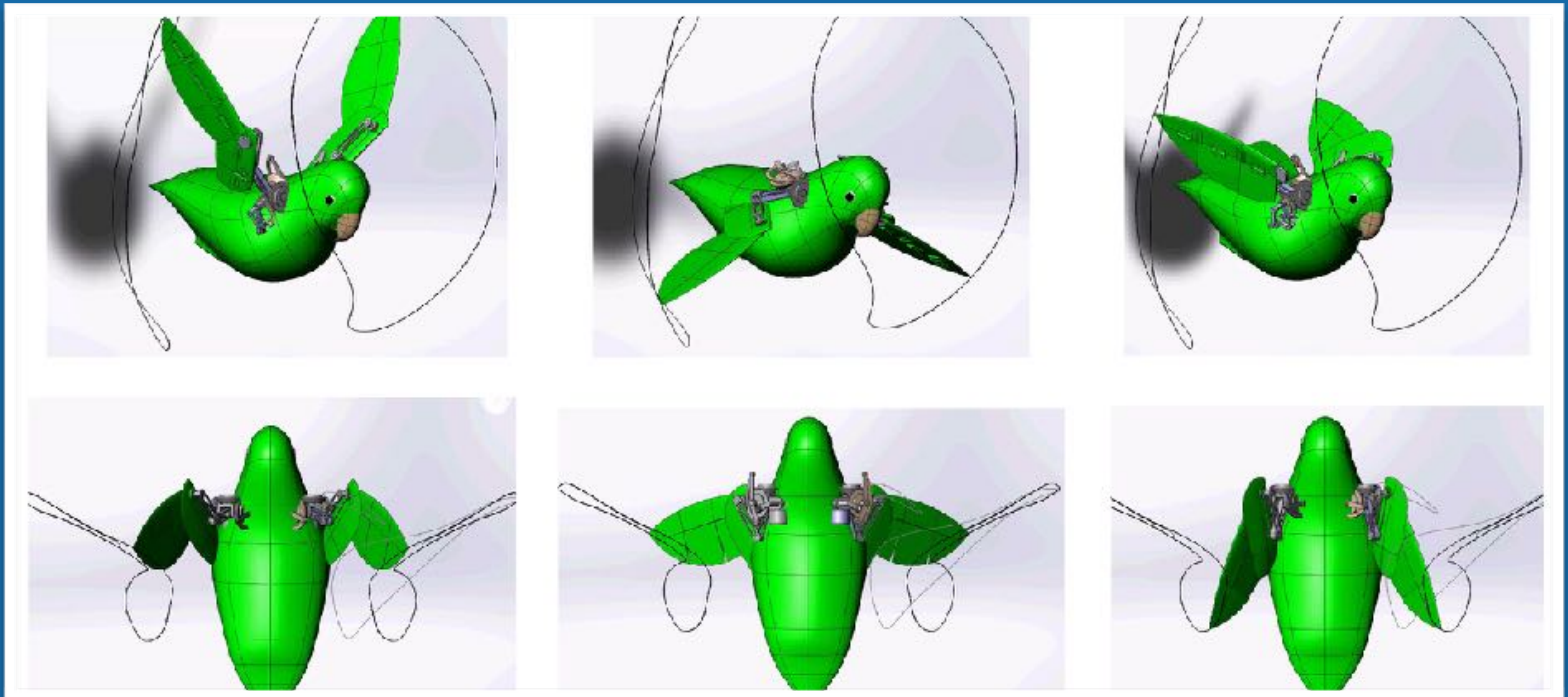
J. Michael McCarthy received his doctorate at Stanford University and taught at Loyola Marymount University and the University of Pennsylvania before joining UCI in 1986.

He has published articles in over 250 publications and five books including *Introduction to Theoretical Kinematics* (MIT Press) and *Geometric Design of Linkages* (Springer 2000, 2nd Ed. 2010). He has served as the editor-in-chief of the *ASME Journal of Mechanical Design* (2002-2007) and is the founding editor-in-chief of the *ASME Journal of Mechanisms and Robotics* (2007-2014).

His research team is responsible for the *Sphinx*, *Synthetica* and *MecGen* software packages, which extend computer-aided design to spherical and spatial linkage systems and integrate this process with geometric modeling. He has organized and presented tutorials on the design of linkages and robotic systems at ASME and IEEE conferences, including the NSF-sponsored 2012 Workshop on 21st Century Kinematics.

He is a fellow of the American Society of Mechanical Engineers (ASME), and has received the 2009 ASME Machine Design Award, the 2011 ASME Mechanisms and Robotics Award, and the 2013 Robert E. Abbott Lifetime Service Award from the Design Engineering Division of ASME International.

At the 2015 Mechanisms and Robotics Conference, he and his co-author received the A.T. Yang Memorial Award in Theoretical Kinematics for their paper on the design of a linkage system that reproduces the flapping motion of a bird in flight.



Computer Aided Design of a Mechanical System to Draw a Curve

J. Michael McCarthy, University of California, Irvine
Sigma Xi Research Society of Orange County
November 19, 2019

Ferdinand Freudenstein,
History of Machines and Mechanisms

Journal of Mechanical Design

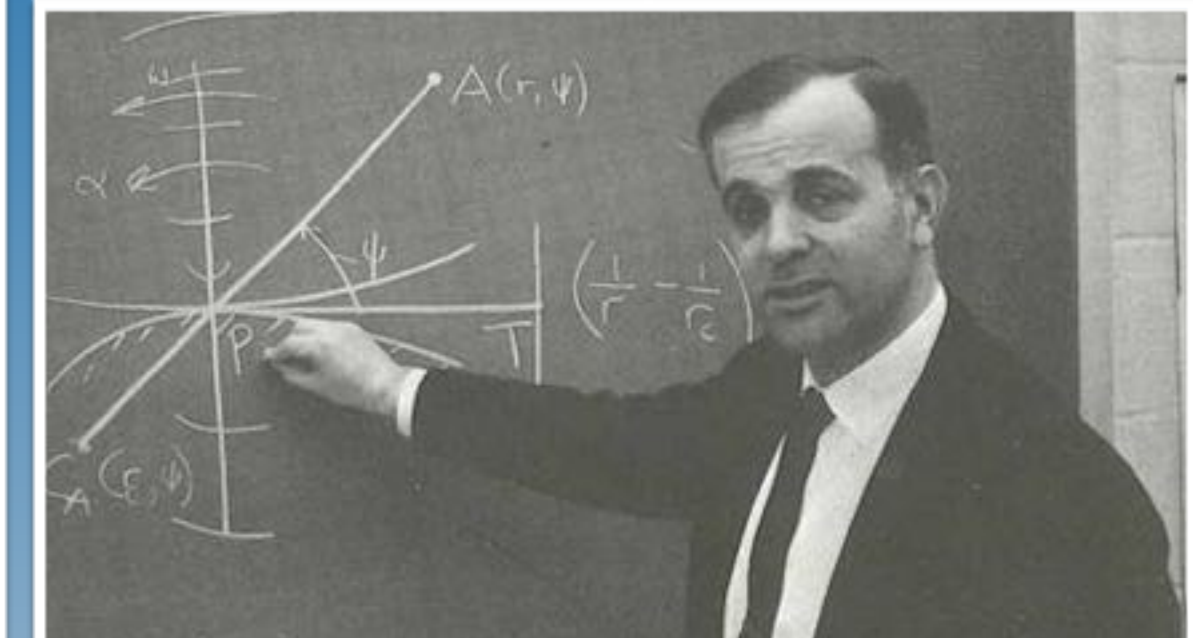
Editorial

In Memory of Professor Ferdinand Freudenstein, 1926–2006

In the spring of 1999 Professor Marco Ceccarelli invited Professor Ferdinand Freudenstein to present the opening plenary lecture at the International Symposium on History of Machines and Mechanisms, sponsored by IFToMM and scheduled to be held in May 2000 in Cassino, Italy.

Professor Ceccarelli requested that the lecture contain a short overview of the field in relation to Professor Freudenstein's personal history. However, Professor Freudenstein had retired from Columbia University three years earlier, and by the time the invitation came his health was such that it would have been difficult for him to travel to Europe. Still he accepted the invitation, and in May 1999, with the assistance of his wife Lydia, and a colleague, Professor Richard Longman of Columbia University, he made a videotape of his plenary lecture. This tape was played at the opening of the conference in Cassino in May 2000. It is the last professional presentation by Professor Freudenstein. The transcript of the talk follows.

Bernard Roth
Stanford, CA

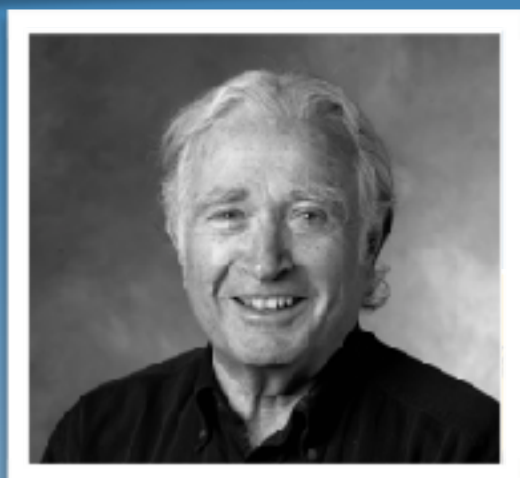


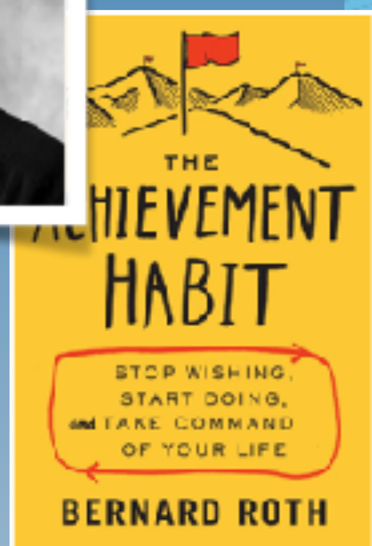
Several decades later, in the 1920s, the "geometrical school" in Germany extended Burmester's ideas and began to formulate, for the first time, the concept of "kinematic synthesis." In contrast to kinematic analysis, which determines the motion characteristics of a given mechanism, kinematic synthesis proceeds from the given motion requirements to the determination of the type and proportions of an appropriate mechanism.

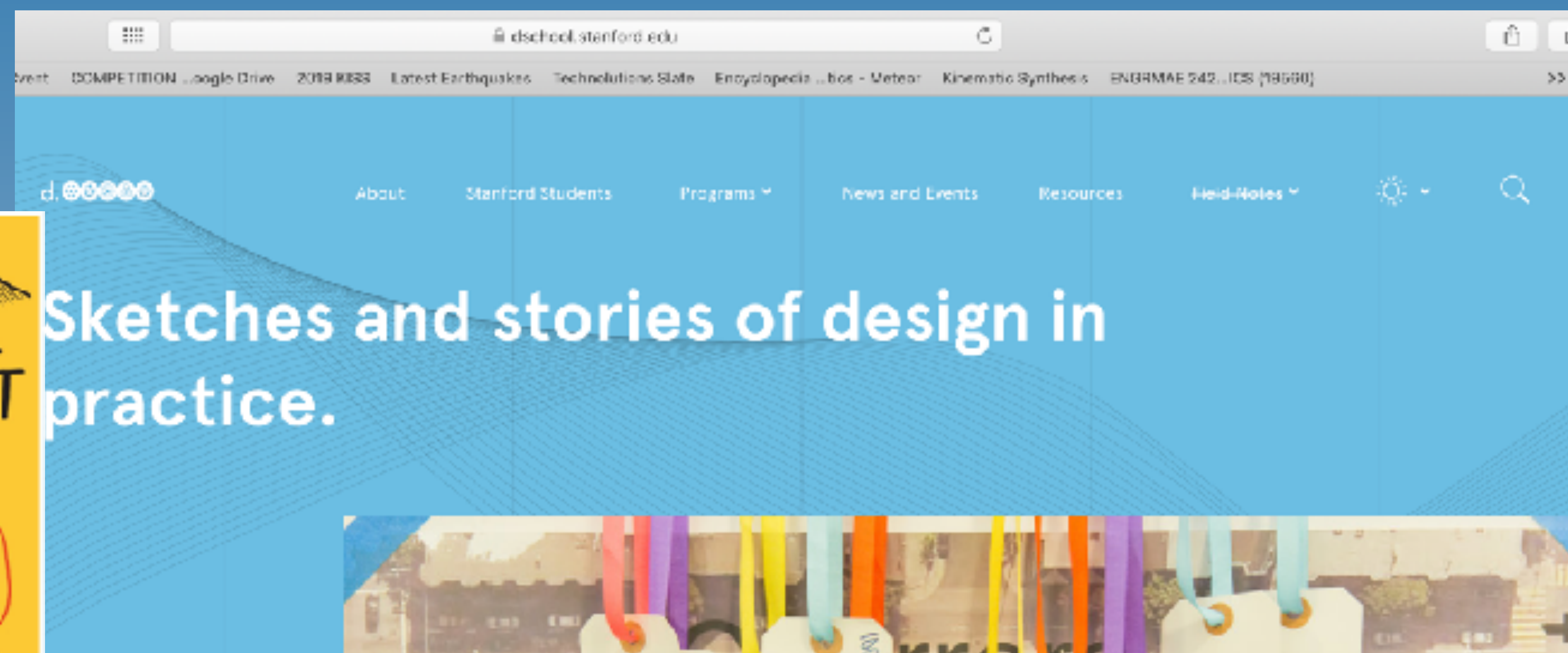
F. Freudenstein, Plenary Lecture, International Symposium on the History of Machines and Mechanisms, May 2000.Journal of Mechanical Design, March 2007:

Kinematic Synthesis and Innovation

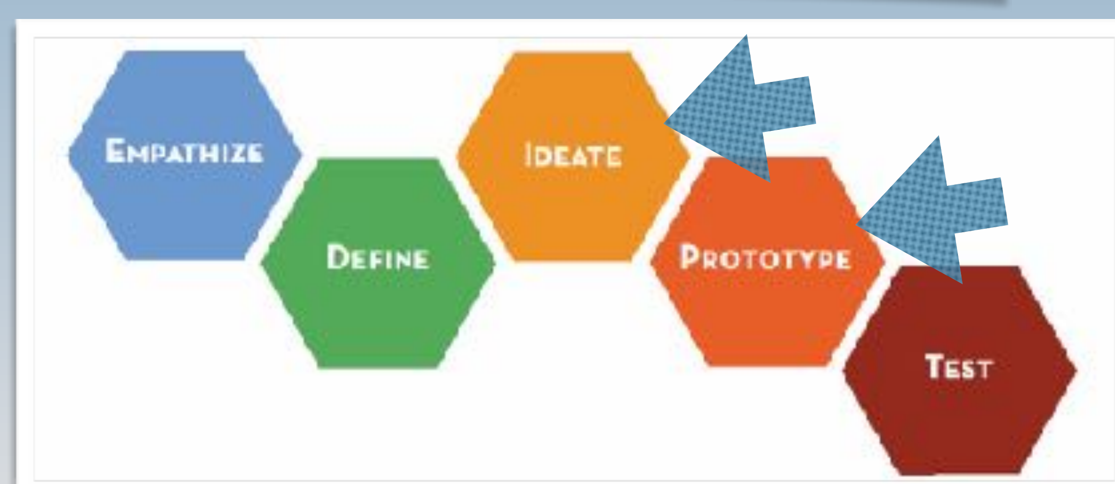
Kinematic synthesis of robotic systems is part of the design process.





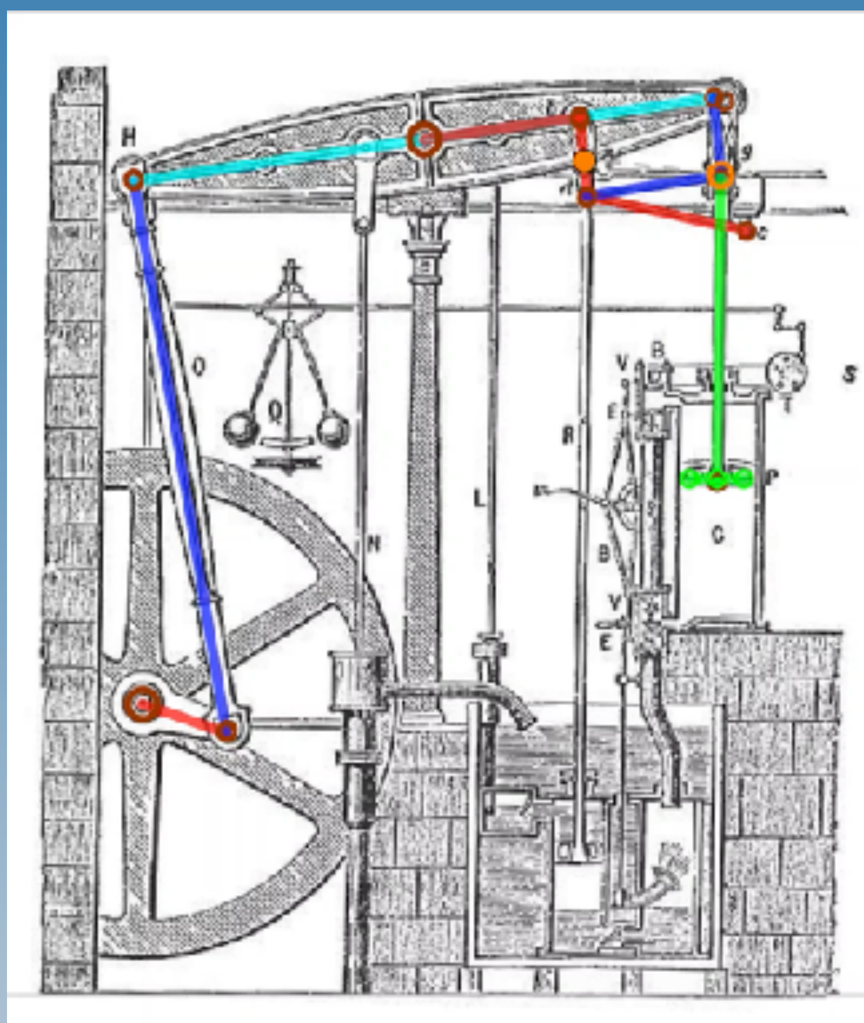


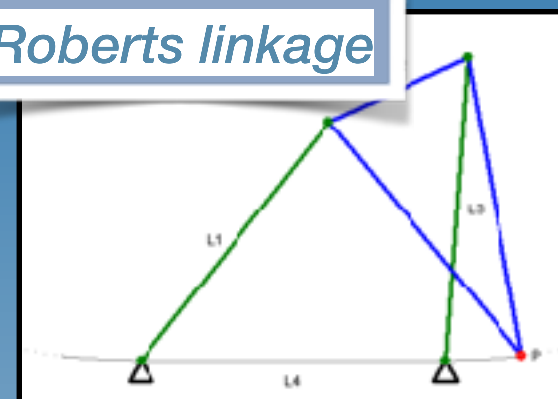
*Institute for Design at Stanford University, **Design Thinking***



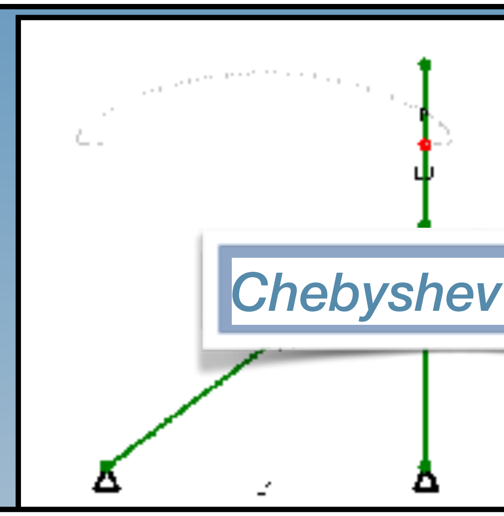
The Steam Engine and Drawing a Straight Line

James Watt (1775) used a straight-line linkage to guide double-acting expansion in the steam engine.

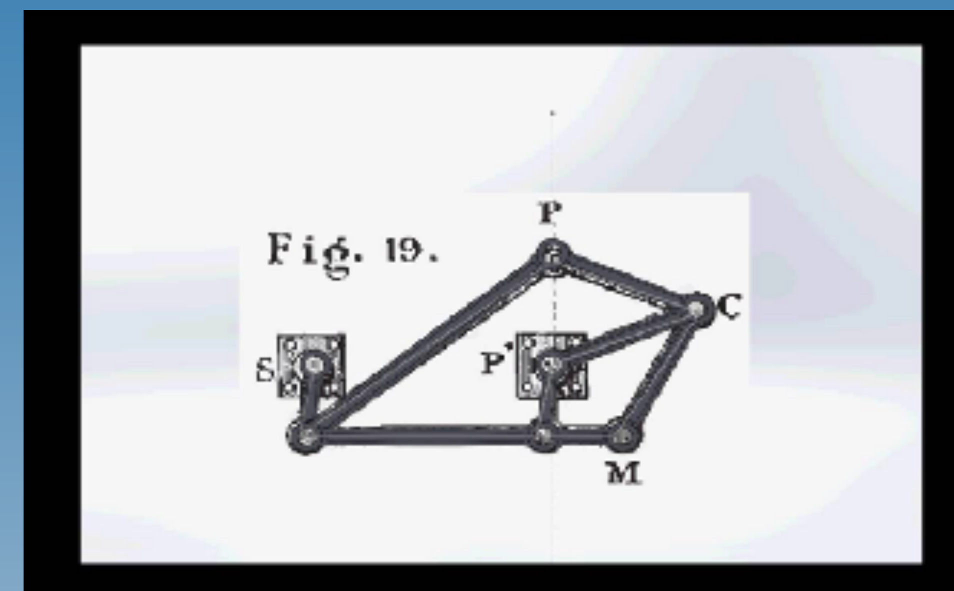




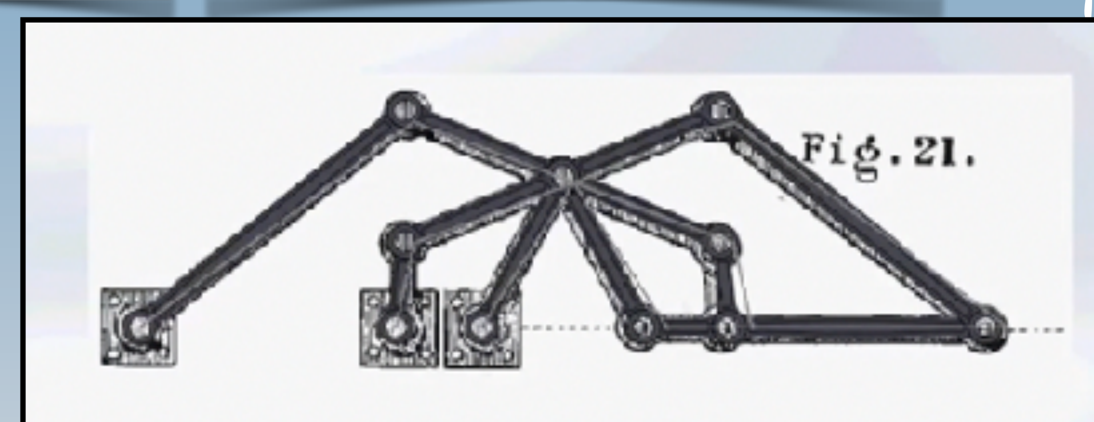
Roberts linkage



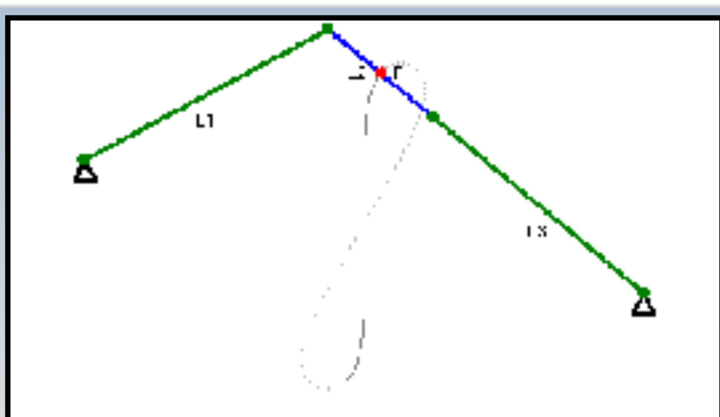
Chebyshev linkage

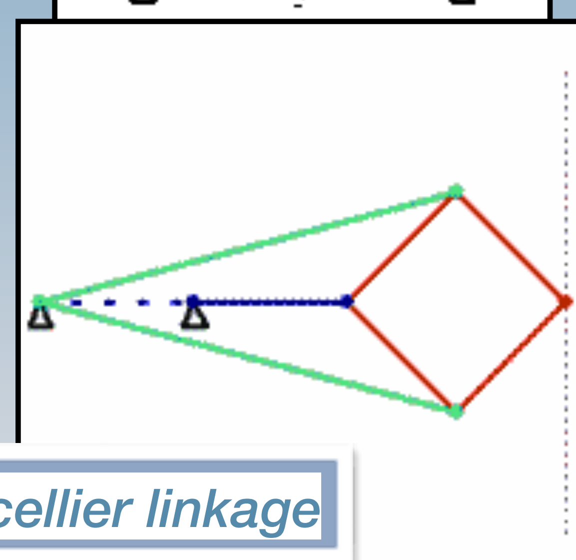


Kempe straight line linkage



Kempe rectilinear linkage





Peaucellier linkage

Linkages Draw Algebraic Curves

Kempe (1876) showed for any algebraic curve there is linkage that draws the curve, Kempe's Universality Theorem.

On a General Method of describing Plane Curves of the n^{th} degree by Linkwork. By A. B. Kempe, B.A.

[*Read June 9th*, 1876.]

Lemma I.—*The Reversor.*—Let $O\xi\beta\alpha$ (Fig. 1) be the linkage known as the contra-parallelogram, $O\xi$ being equal to $\beta\alpha$, and $O\alpha$ to $\xi\beta$.

Make $\alpha\gamma$ a third proportional to $O\xi$ and $O\alpha$, and add the links $O\delta$, equal to $\alpha\gamma$, and $\delta\gamma$, equal to $O\alpha$.

Then the figure $O\alpha\gamma\delta$ is a contra-parallelogram similar to $O\xi\beta\alpha$; and the angle $\xi O\alpha$ is equal to the angle $\delta O\alpha$.

Thus, if $O\xi$ be made to make any angle with $O\alpha$, $O\delta$ will make the same angle with $O\alpha$ on the other side of it.*

negate

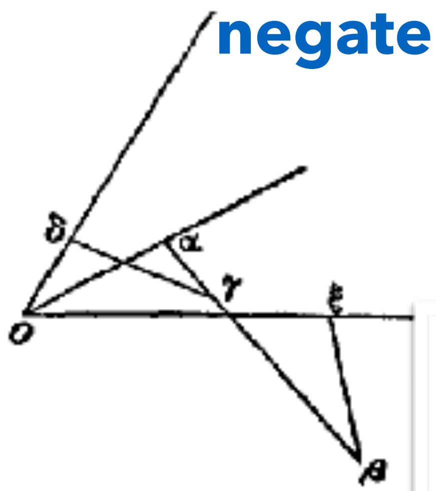


Fig. 1.

* This linkage, and the one next described, were first given by me in the senger of Mathematics," Vol. IV., pp. 122, 123, in a paper "On some new ages," §§ 4 and 8.

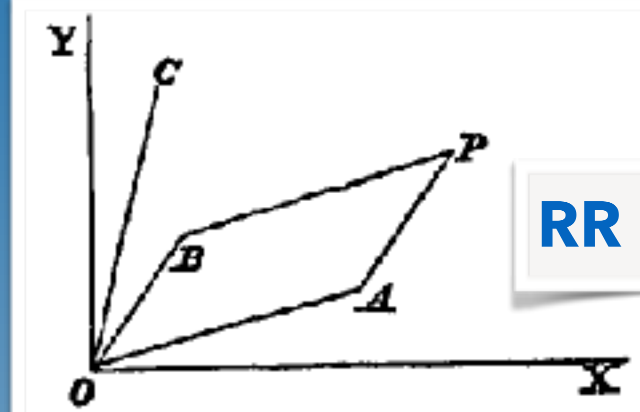


Fig. 5.

RR chain

$$f(x,y) = \sum[A\cos(r\phi \pm s\theta \pm a)] + C = 0$$

multiply

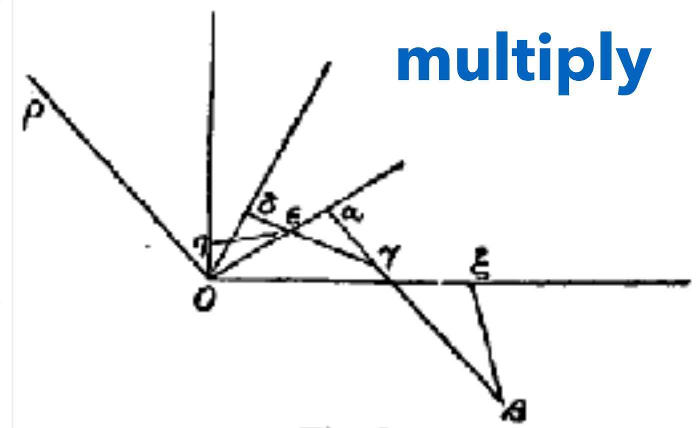


Fig. 2.

add

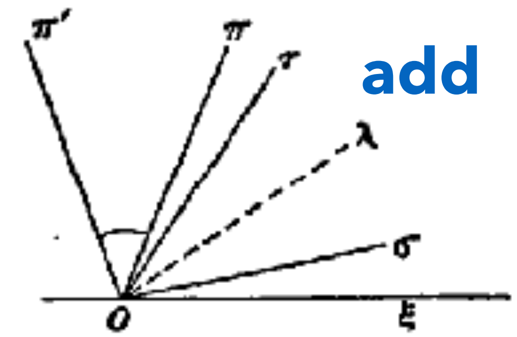


Fig. 3.

translate

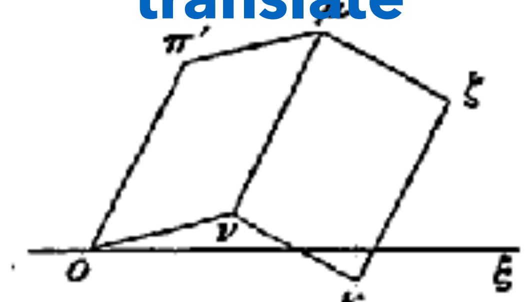
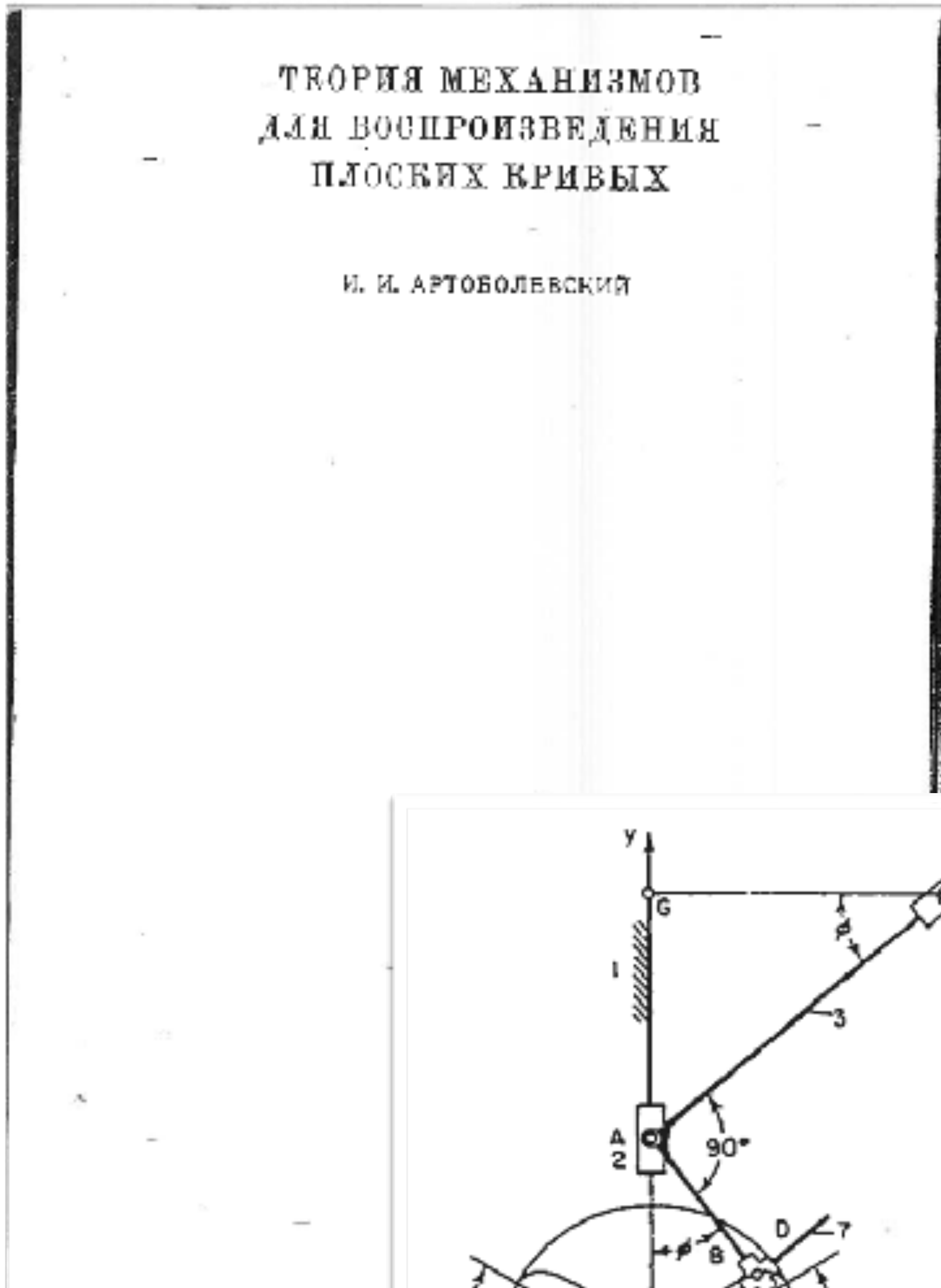


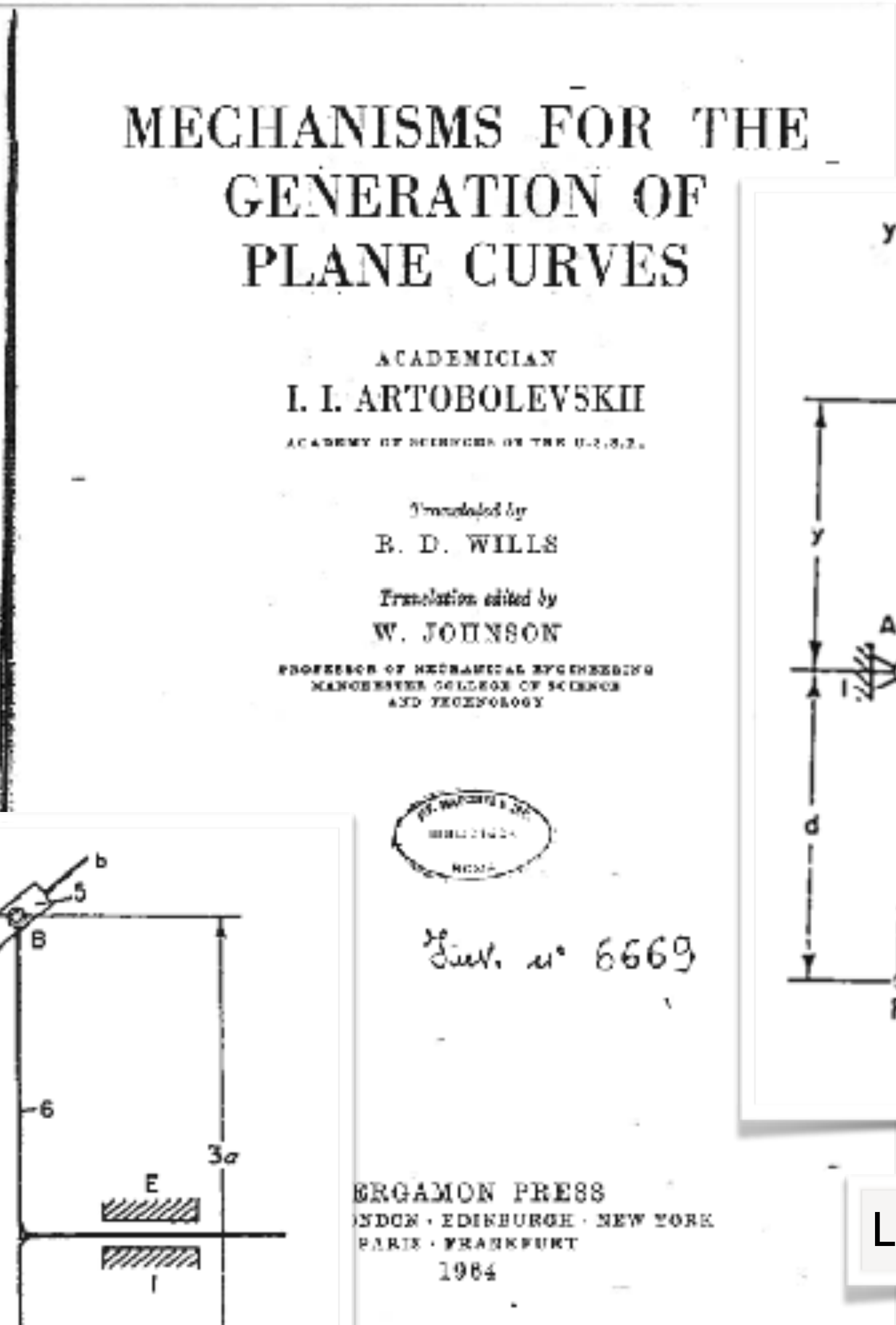
Fig. 4.

The method has, however, an interest, as showing that there is a way of drawing any given case; and the variety of methods of expressing

Drawing Plane Curves


ТЕОРИЯ МЕХАНИЗМОВ
ДЛЯ ВОСПРОИЗВЕДЕНИЯ
ПЛОСКИХ КРИВЫХ

И. И. АРТОБОЛЕВСКИЙ


MECHANISMS FOR THE
GENERATION OF
PLANE CURVES

ACADEMICIAN
I. I. ARTOBOLEVSKII
ACADEMY OF SCIENCES OF THE U.S.S.R.

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MANCHESTER COLLEGE OF SCIENCE
AND TECHNOLOGY

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PARIS · FRANKFURT
1964

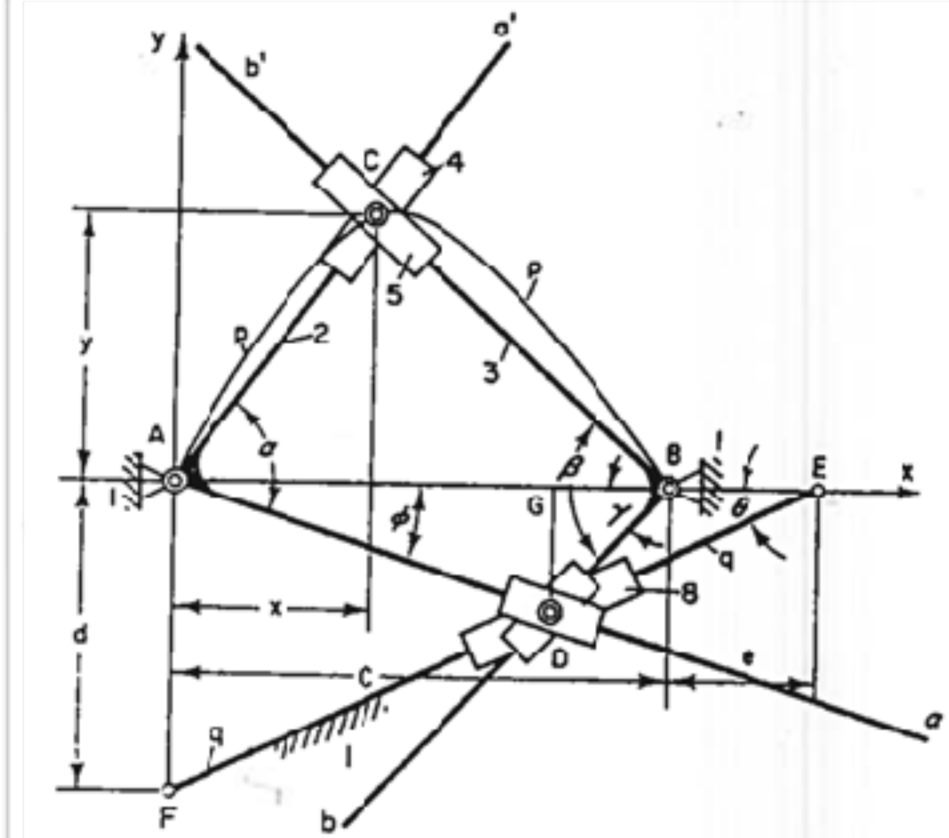
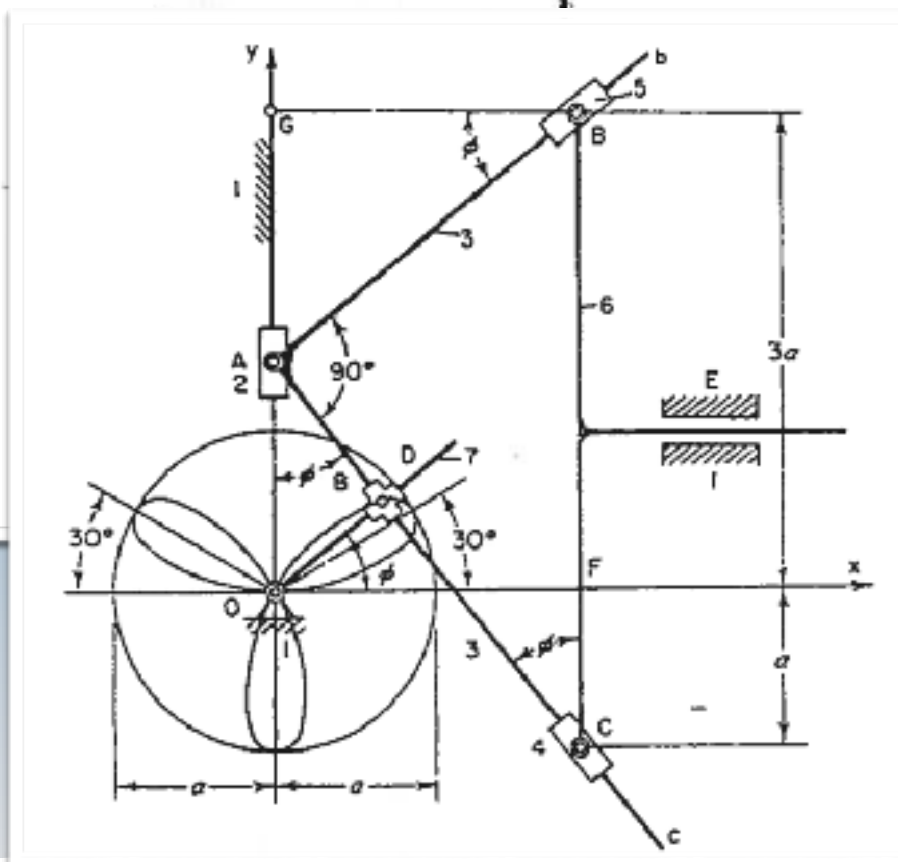


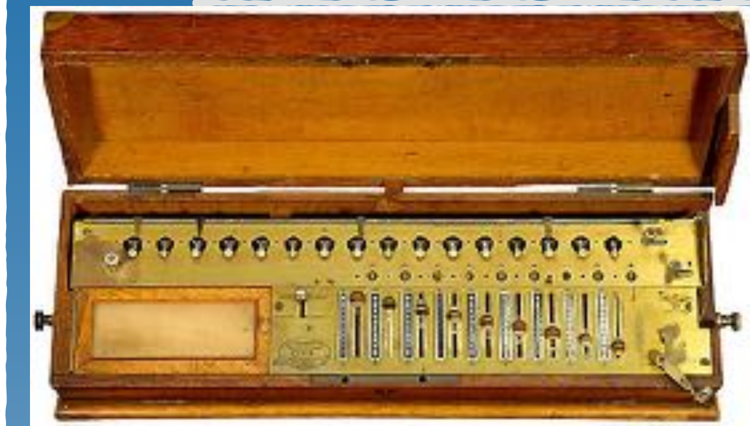
FIG. 136

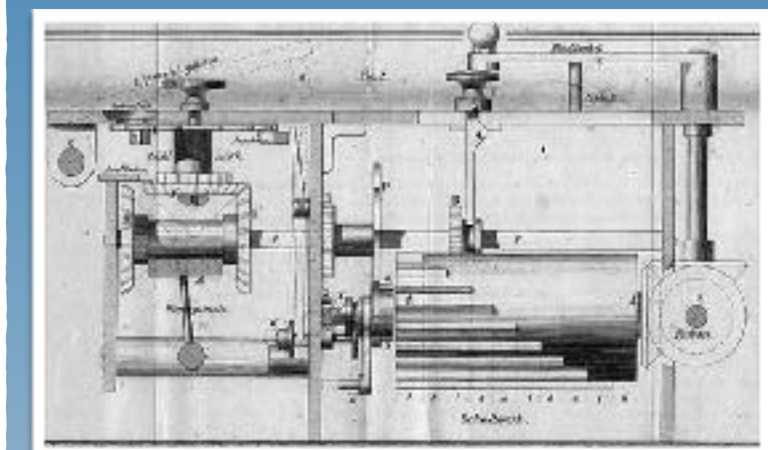
Linkage to trace conic sections.



Linkage to trace a trifolium

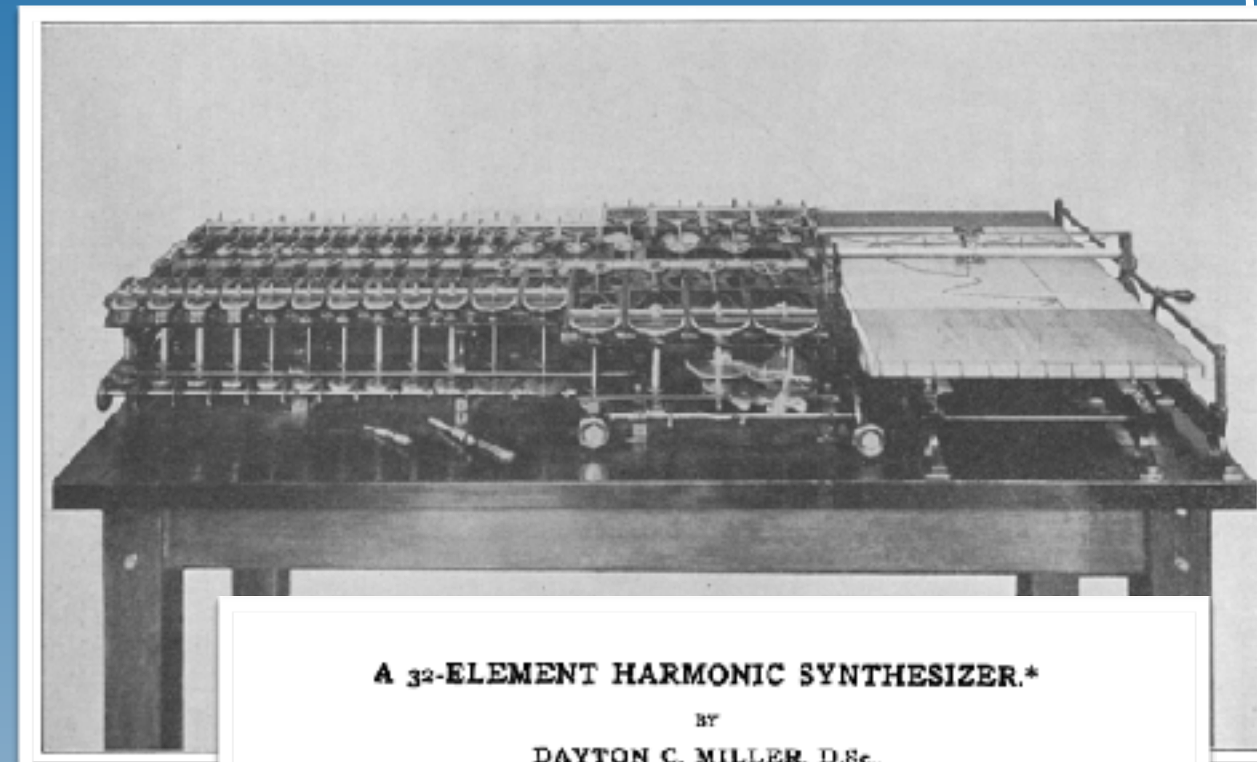
Mechanical Computation

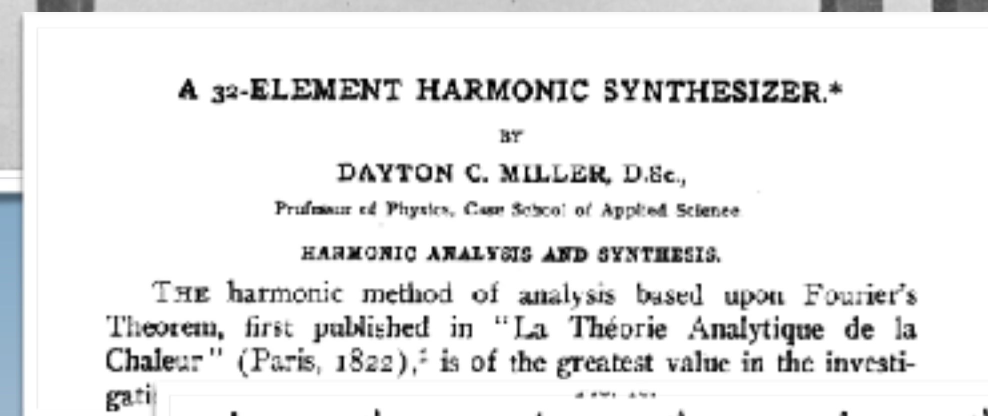




1820 Arithmometer, Thomas de Colmar was the first commercial calculator.

1887 Comptometer, Dorr E. Felt was the first keyed calculator.





A 32-ELEMENT HARMONIC SYNTHESIZER.*

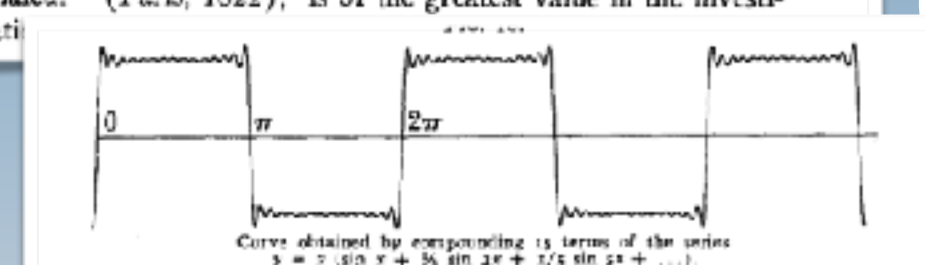
BY

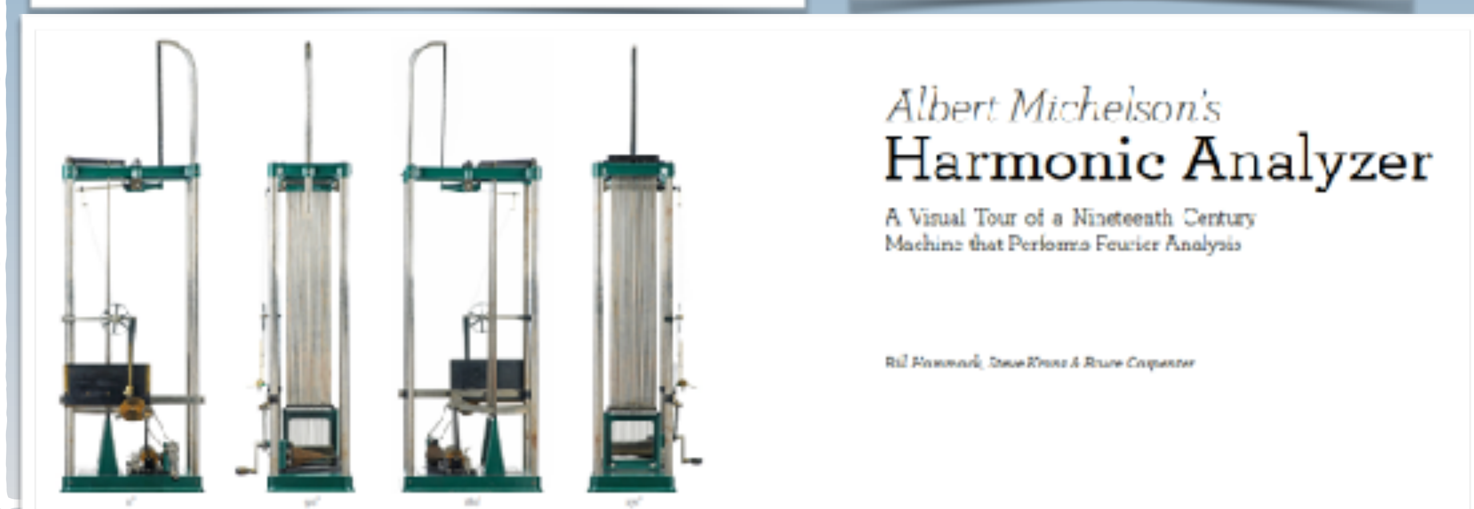
DAYTON C. MILLER, D.Sc.,

Professor of Physics, Case School of Applied Science

HARMONIC ANALYSIS AND SYNTHESIS.

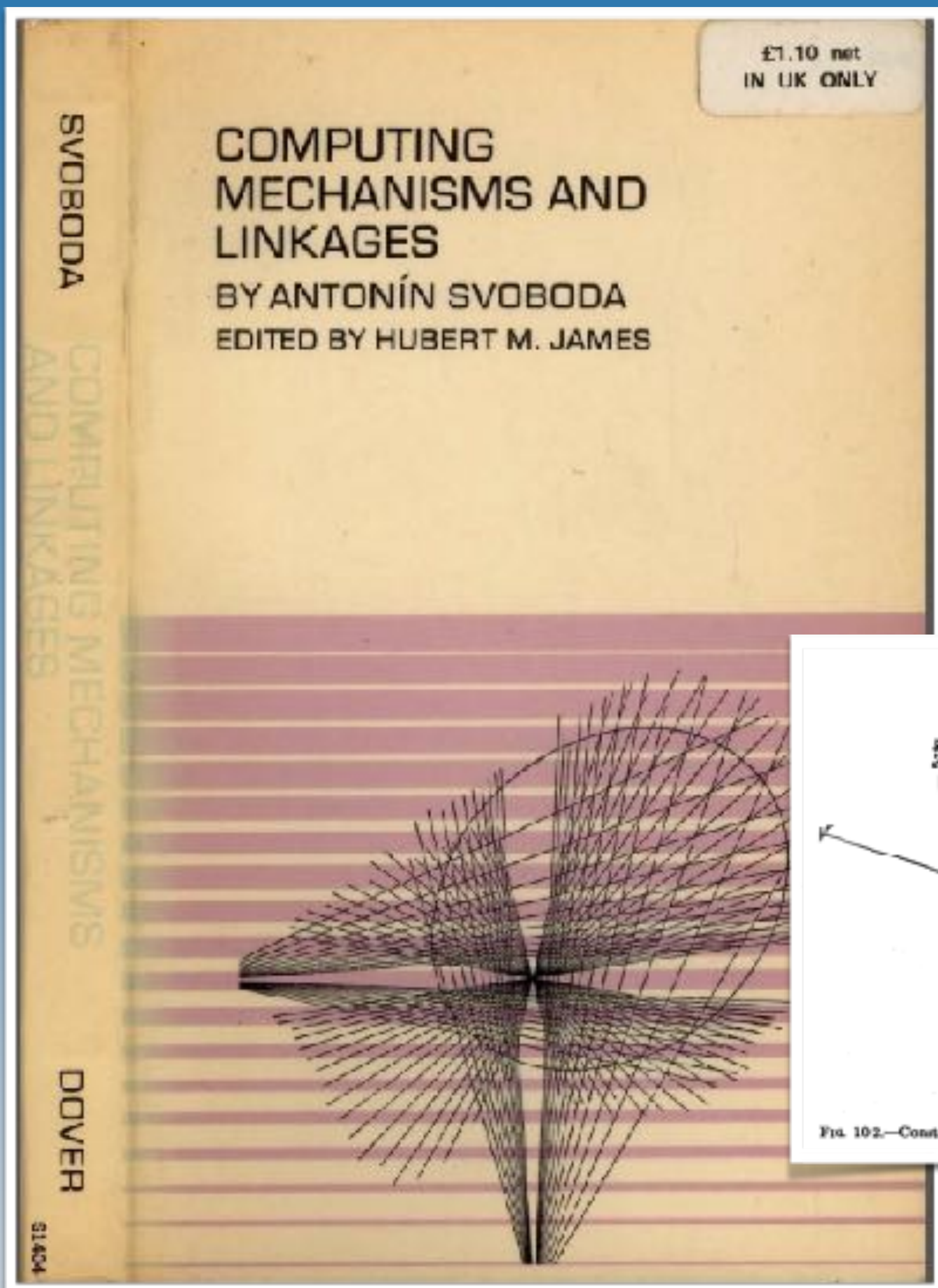
The harmonic method of analysis based upon Fourier's Theorem, first published in "La Théorie Analytique de la Chaleur" (Paris, 1822),[1] is of the greatest value in the investi- gati

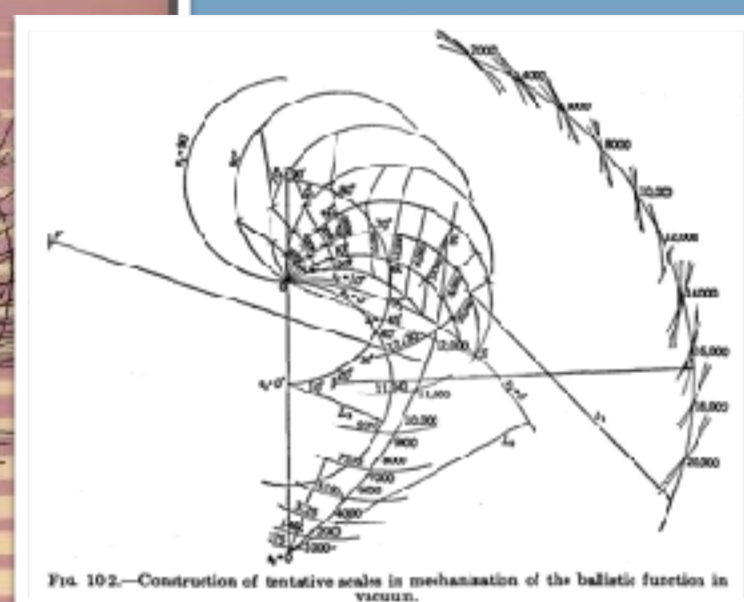


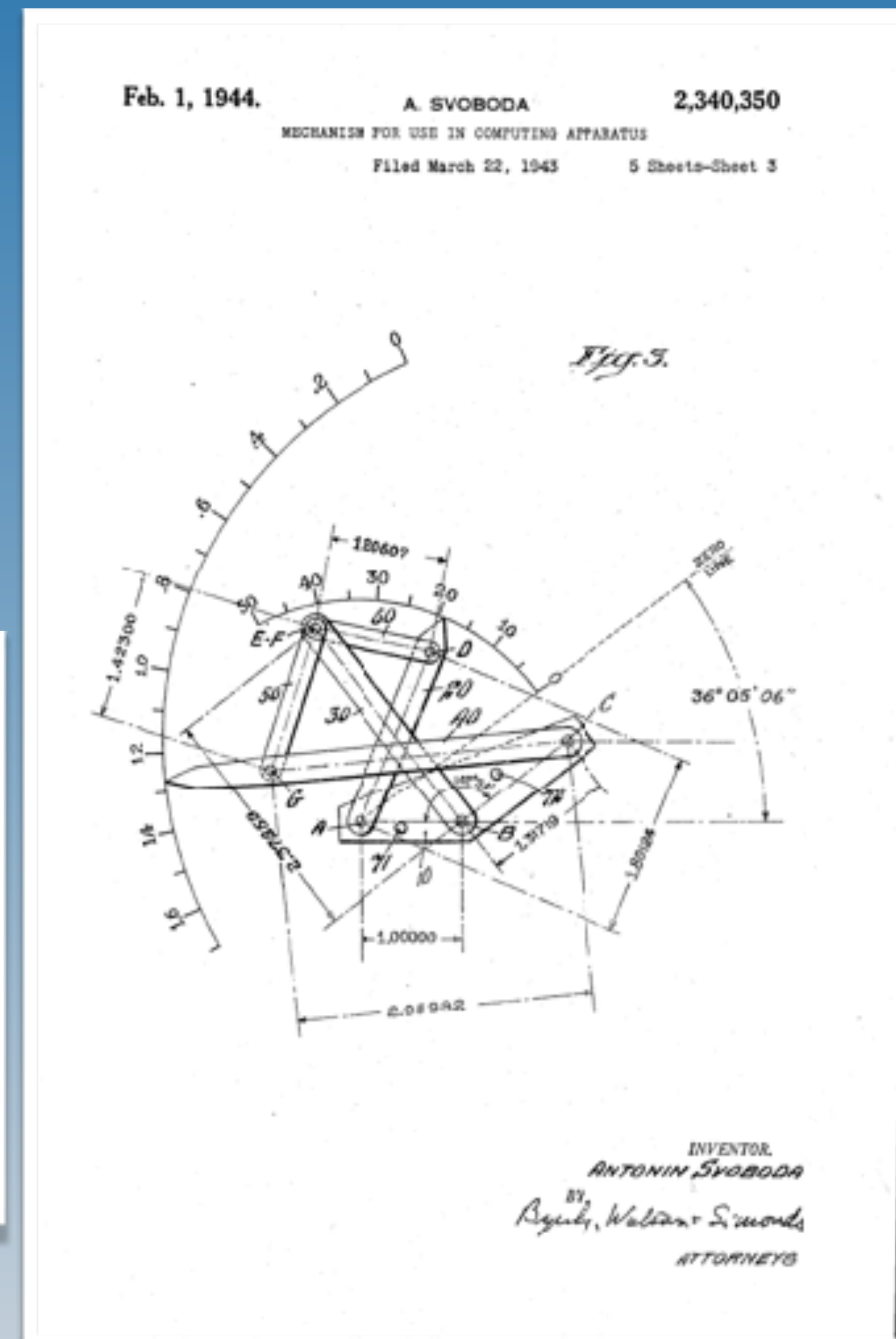


D. C. Miller, ``A 32-element Harmonic Synthesizer,'' Journal of the Franklin Institute, Jan. 1916.

Geometric Design



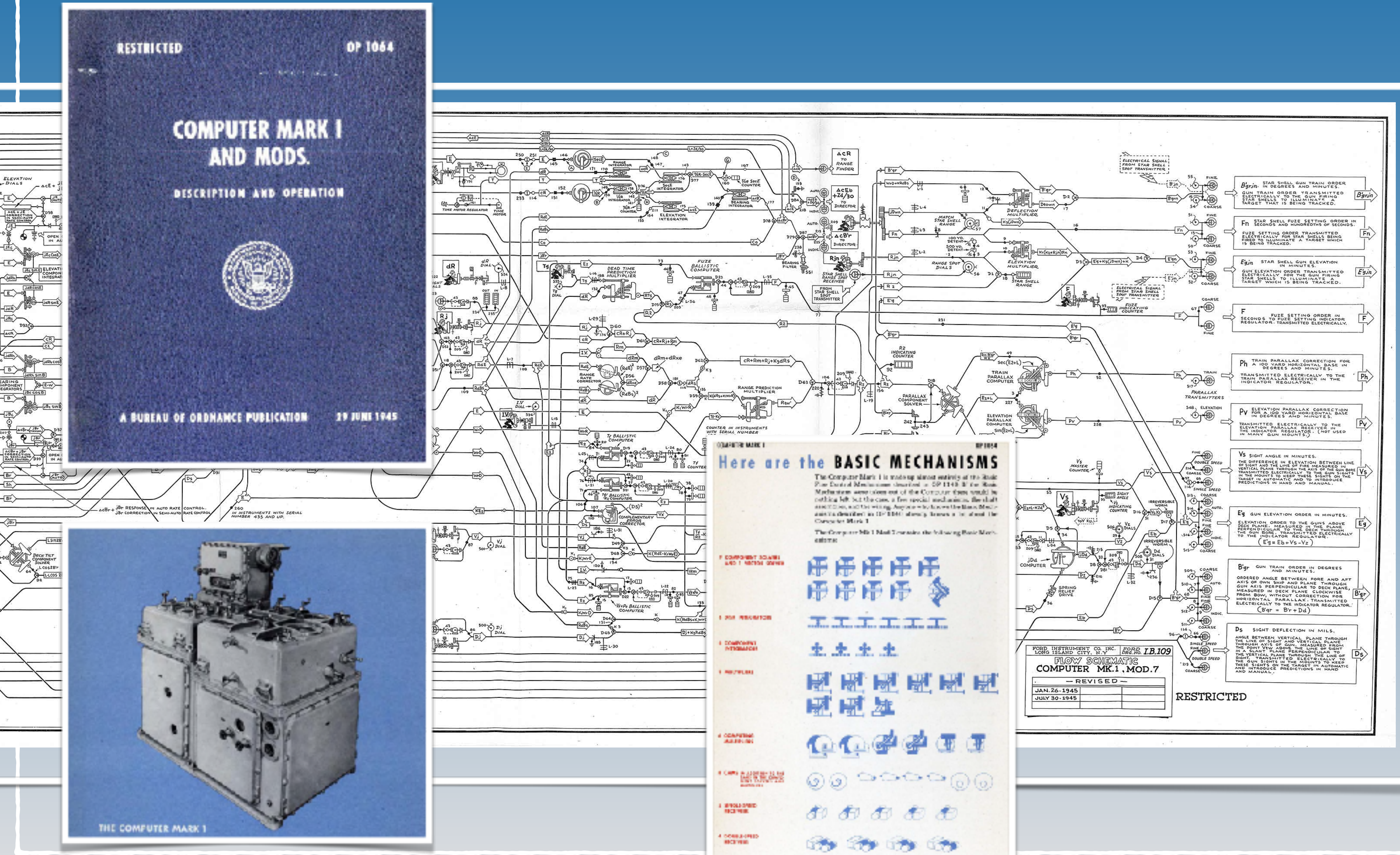




A. Svoboda, Computing Machines and Linkages, McGraw-Hill, 1948.

A. Svoboda, Six-bar linkage function generator to compute logarithms. US Patent No. 2,340,350

Mechanical Computer

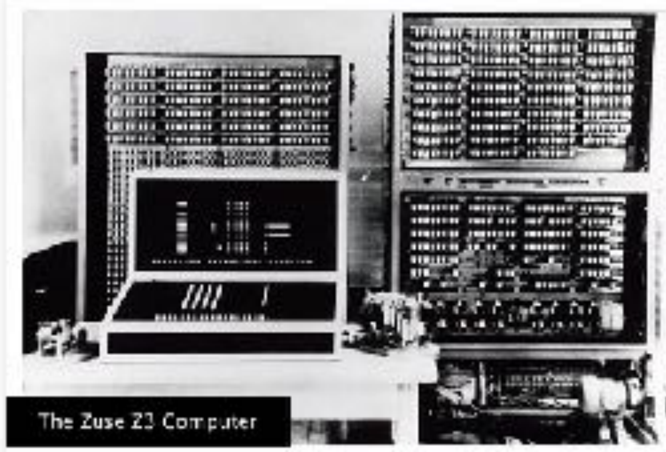


Early Digital Computers

1941	1942	1944

1941

Konrad Zuse finishes the Z3 Computer


The Zuse Z3 Computer

The Z3, an early computer built by German engineer Konrad Zuse working in complete isolation from developments elsewhere, uses 2,300 relays, performs floating point binary arithmetic, and has a 22-bit word length. The Z3 was used for aerodynamic calculations but was destroyed in a bombing raid on Berlin in late 1943. Zuse later supervised a reconstruction of the Z3 in the 1960s, which is currently on display at the Deutsches Museum in Munich.

1942

The Atanasoff-Berry Computer (ABC) is completed


The Atanasoff-Berry Computer

After successfully demonstrating a proof-of-concept prototype in 1939, Professor John Vincent Atanasoff receives funds to build a full-scale machine at Iowa State College (now University). The machine was designed and built by Atanasoff and graduate student Clifford Berry between 1939 and 1942. The ABC was at the center of a patent dispute related to the invention of the computer,

1944

Harvard Mark 1 is completed


Harvard Mark 1 is completed

Conceived by Harvard physics professor Howard Aiken, and designed and built by IBM, the Harvard Mark 1 is a room-sized, relay-based calculator. The machine had a fifty-foot long camshaft running the length of machine that synchronized the machine's thousands of component parts and used 3,500 relays. The Mark 1 produced mathematical tables but was soon superseded by electronic stored-program computers.

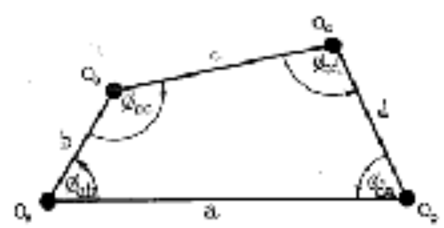
source: ComputerHistory.org

Freudenstein's Approach to Design

An Analytical Approach to the Design of Four-Link Mechanisms[1]

By FERDINAND FREUDENSTEIN,[2] NEW YORK, N. Y.

The symmetry inherent in four-link mechanisms has been used in deriving a general analytical expression relating the diagonals of a four-link mechanism by means of a characteristic parameter. The nature of this expression has been analyzed. The particular case obtained by using as a parameter the square root of the sum of the squares of the lengths of the links diminished by the sum of the squares of the diagonals, has been worked out in detail and the results reduced to a form believed to be useful in design work. Design applications have been considered.



Approximate Synthesis of Four-Bar Linkages[1,2]

By FERDINAND FREUDENSTEIN,[3] NEW YORK, N. Y.

Formulas are presented for obtaining the characteristics of a four-bar linkage, designed to generate an arbitrary function approximately over a finite range. A number of methods of varying degrees of accuracy and complexity have been developed, enabling a designer to select the one best suited to his requirements.

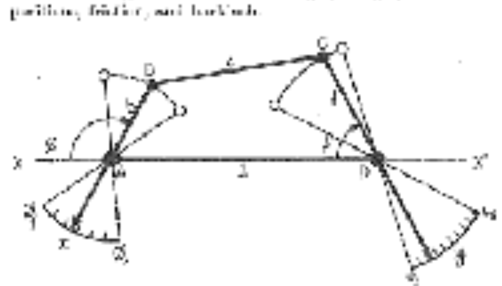


FIG. 1 FOUR-BAR LINKAGE

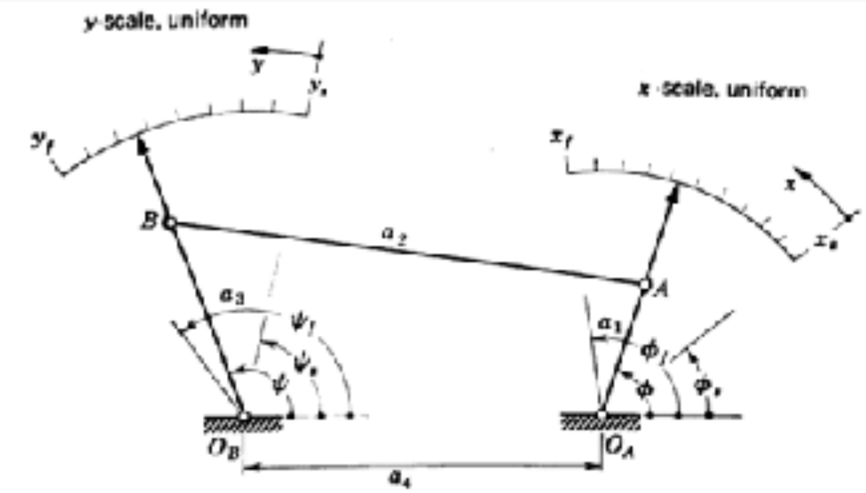


FIGURE 10-3 Principle of four-bar-linkage function generator.

10-2 CRANK AND FOLLOWER SYNTHESIS: THREE ACCURACY POINTS

Consider the problem of designing a planar four-bar linkage such that to three given positions of the crank, defined by angles ϕ_1, ϕ_2, and ϕ_3, there correspond three prescribed positions of the follower, ψ_1, ψ_2, and ψ_3. The solution consists in finding the proper values of a_1, a_2, a_3, and a_4 for three related pairs (ϕ_1, ψ_1), (ϕ_2, ψ_2), and (ϕ_3, ψ_3). The procedure is based on the displacement equation[1]

$$K_1 \cos\phi - K_2 \cos\psi + K_3 = \cos(\phi - \psi) \qquad (10\text{-}3)$$

with $K_1 = \dfrac{a_4}{a_3} \qquad K_2 = \dfrac{a_4}{a_1} \qquad K_3 = \dfrac{a_1^2 - a_2^2 + a_3^2 + a_4^2}{2a_1a_3}$

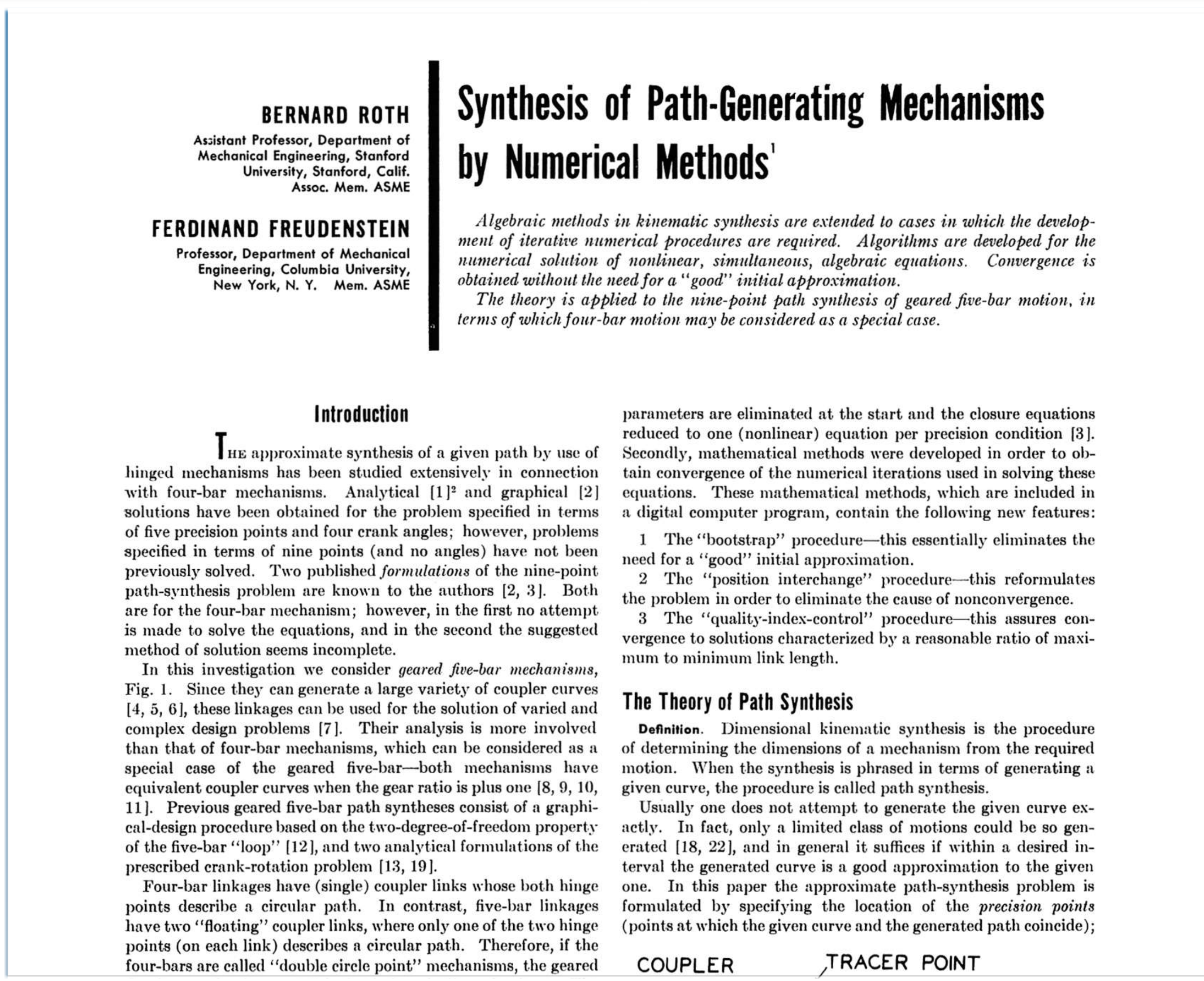
This equation was deduced from Eq. (10-1) by rearranging the terms. When written for three pairs of values, (ϕ_1, ψ_1), (ϕ_2, ψ_2), (ϕ_3, ψ_3), this equation yields a system of three equations linear with respect to K_1, K_2, K_3.

$$\begin{aligned} K_1 \cos\phi_1 - K_2 \cos\psi_1 + K_3 &= \cos(\phi_1 - \psi_1) \\ K_1 \cos\phi_2 - K_2 \cos\psi_2 + K_3 &= \cos(\phi_2 - \psi_2) \\ K_1 \cos\phi_3 - K_2 \cos\psi_3 + K_3 &= \cos(\phi_3 - \psi_3) \end{aligned}$$

[1] This is also known as the Freudenstein equation (see first reference in Bibliography at the end of this chapter).

R. S. Hartenberg and J. Denavit, Kinematic Synthesis of Linkages, McGraw-Hill, 1964.

Computer Aided Design



Synthesis of Path-Generating Mechanisms by Numerical Methods[1]

BERNARD ROTH
Assistant Professor, Department of Mechanical Engineering, Stanford University, Stanford, Calif. Assoc. Mem. ASME

FERDINAND FREUDENSTEIN
Professor, Department of Mechanical Engineering, Columbia University, New York, N. Y. Mem. ASME

Algebraic methods in kinematic synthesis are extended to cases in which the development of iterative numerical procedures are required. Algorithms are developed for the numerical solution of nonlinear, simultaneous, algebraic equations. Convergence is obtained without the need for a "good" initial approximation.

The theory is applied to the nine-point path synthesis of geared five-bar motion, in terms of which four-bar motion may be considered as a special case.

Introduction

THE approximate synthesis of a given path by use of hinged mechanisms has been studied extensively in connection with four-bar mechanisms. Analytical [1][2] and graphical [2] solutions have been obtained for the problem specified in terms of five precision points and four crank angles; however, problems specified in terms of nine points (and no angles) have not been previously solved. Two published *formulations* of the nine-point path-synthesis problem are known to the authors [2, 3]. Both are for the four-bar mechanism; however, in the first no attempt is made to solve the equations, and in the second the suggested method of solution seems incomplete.

In this investigation we consider *geared five-bar mechanisms*, Fig. 1. Since they can generate a large variety of coupler curves [4, 5, 6], these linkages can be used for the solution of varied and complex design problems [7]. Their analysis is more involved than that of four-bar mechanisms, which can be considered as a special case of the geared five-bar—both mechanisms have equivalent coupler curves when the gear ratio is plus one [8, 9, 10, 11]. Previous geared five-bar path syntheses consist of a graphical-design procedure based on the two-degree-of-freedom property of the five-bar "loop" [12], and two analytical formulations of the prescribed crank-rotation problem [13, 19].

Four-bar linkages have (single) coupler links whose both hinge points describe a circular path. In contrast, five-bar linkages have two "floating" coupler links, where only one of the two hinge points (on each link) describes a circular path. Therefore, if the four-bars are called "double circle point" mechanisms, the geared parameters are eliminated at the start and the closure equations reduced to one (nonlinear) equation per precision condition [3]. Secondly, mathematical methods were developed in order to obtain convergence of the numerical iterations used in solving these equations. These mathematical methods, which are included in a digital computer program, contain the following new features:

1 The "bootstrap" procedure—this essentially eliminates the need for a "good" initial approximation.

2 The "position interchange" procedure—this reformulates the problem in order to eliminate the cause of nonconvergence.

3 The "quality-index-control" procedure—this assures convergence to solutions characterized by a reasonable ratio of maximum to minimum link length.

The Theory of Path Synthesis

Definition. Dimensional kinematic synthesis is the procedure of determining the dimensions of a mechanism from the required motion. When the synthesis is phrased in terms of generating a given curve, the procedure is called path synthesis.

Usually one does not attempt to generate the given curve exactly. In fact, only a limited class of motions could be so generated [18, 22], and in general it suffices if within a desired interval the generated curve is a good approximation to the given one. In this paper the approximate path-synthesis problem is formulated by specifying the location of the *precision points* (points at which the given curve and the generated path coincide);

COUPLER TRACER POINT

F. Freudenstein and G. N. Sandor, "Synthesis of Path Generating Mechanisms by Means of a Programmed Digital Computer," ASME Journal of Engineering for Industry, 81:159-168, 1959.

B. Roth and F. Freudenstein, "Synthesis of Path-Generating Mechanisms by Numerical Methods," ASME Journal of Engineering for Industry, 85:298-304, 1963.

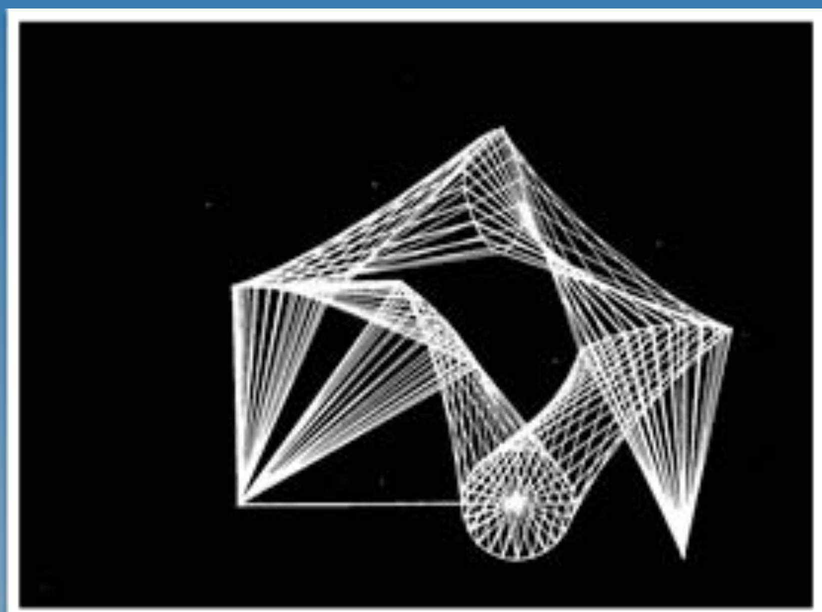
IBM 7090 digital computer: $3m, 36bit, 32k core memory

Roger Kaufman using interactive computer graphics for linkage synthesis at MIT in 1970.

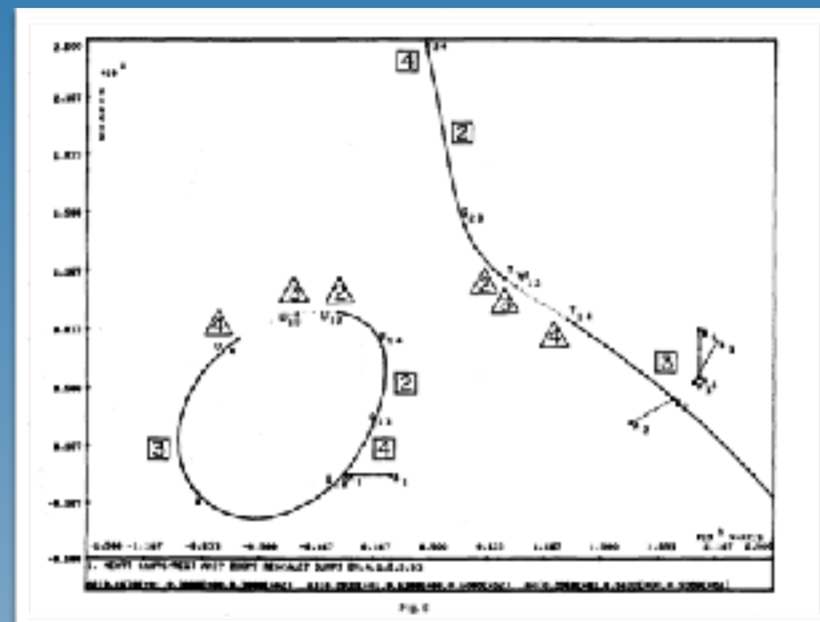
R. E. Kaufman and W. G. Maurer, "Interactive Linkage Synthesis on a Small Computer", ACM National Conference, Aug. 3-5, 1971

IBM 1130 digital computer: $32k, 16bit, 8k core memory

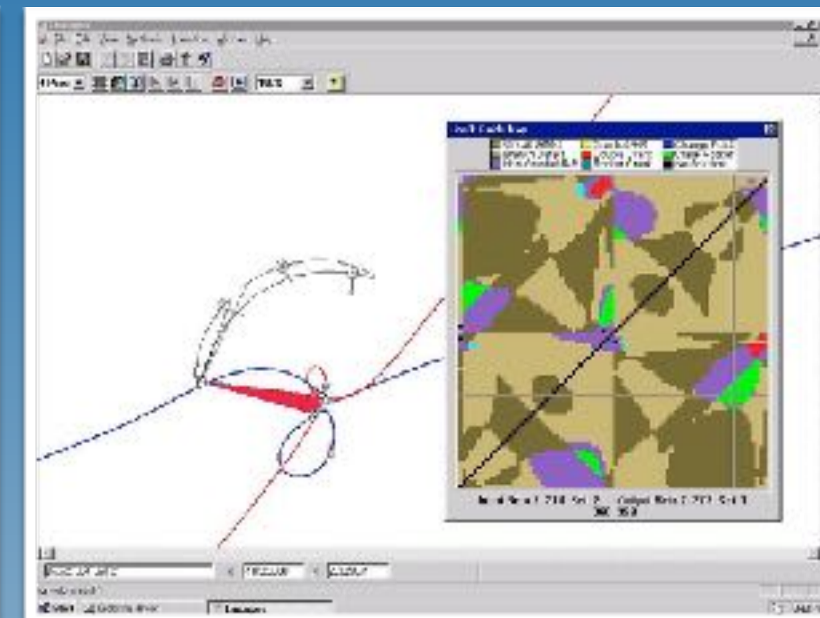
Linkage Synthesis Software



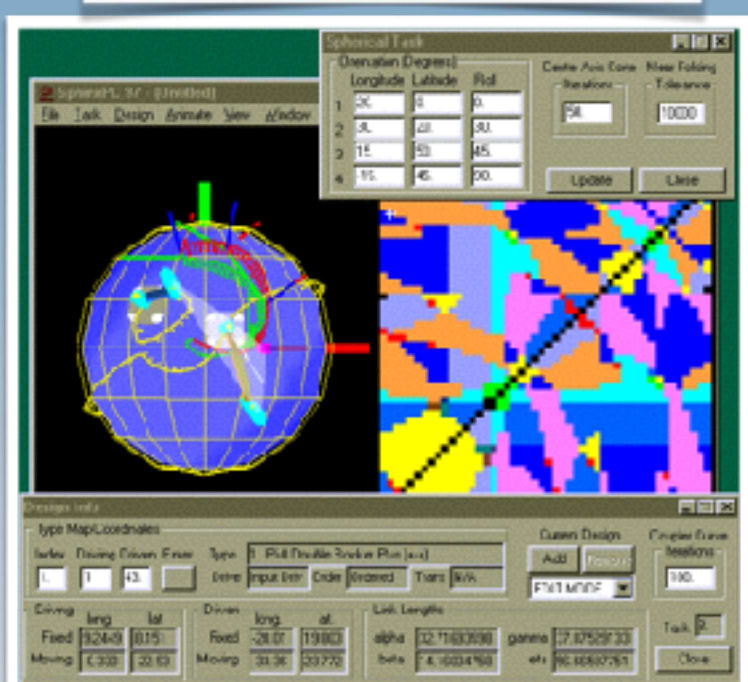
Kinsyn graphical output 1971



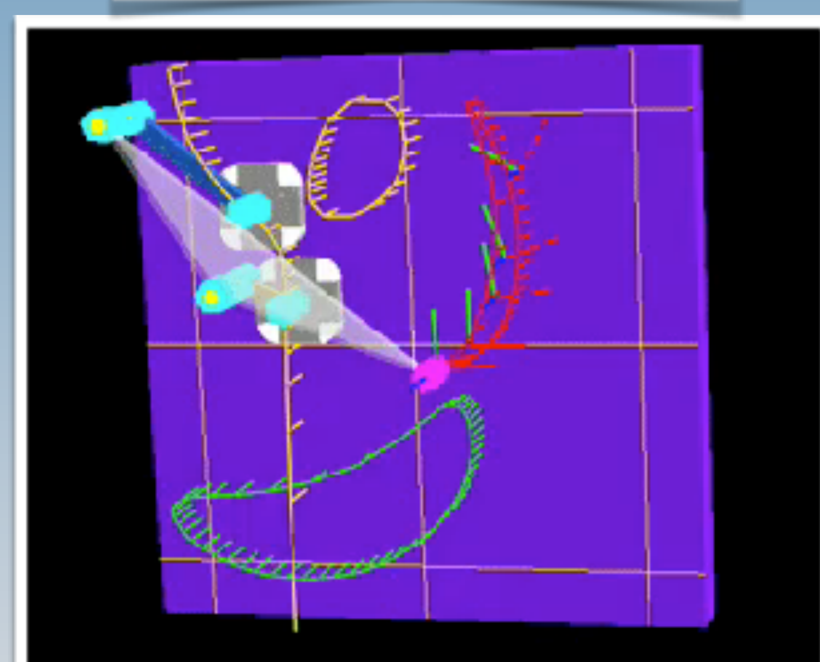
Recsyn graphical output 1981



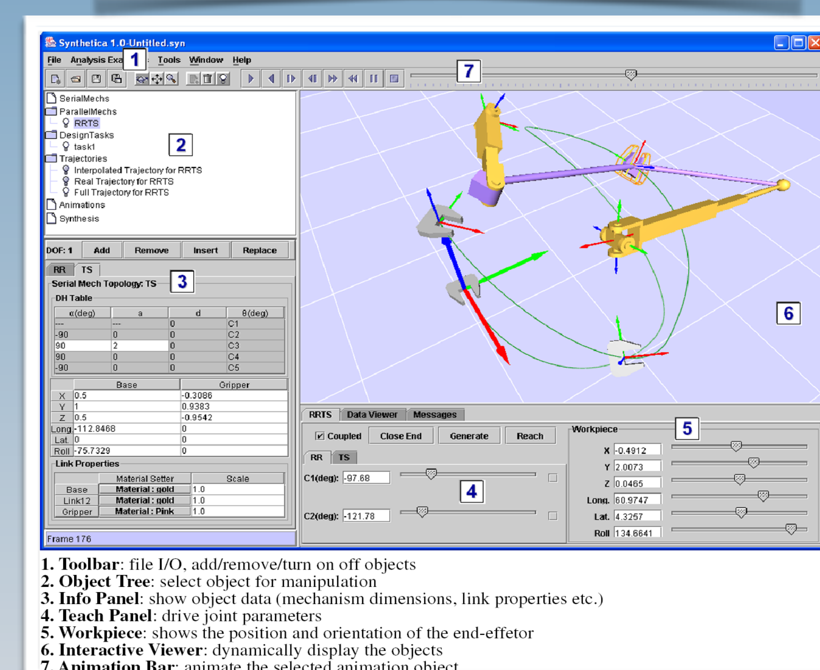
Lincages-4 graphical output 1985



SphinxPC: Spherical 4R design1997



SphinxPC: Planar 4R design 1997



Synthetica: Spatial linkage synthesis 2005

Kempe's Universality Theorem

*Mathematicians Michael Kapovich and John Millson provided what is recognized as the first complete proof of **Kempe's Universality Theorem***

PERGAMON

Topology 41 (2002) 1051–1107

TOPOLOGY

www.elsevier.com/locate/top

Universality theorems for configuration of planar linkages

Michael Kapovich[a,*], John J. Millson[b]

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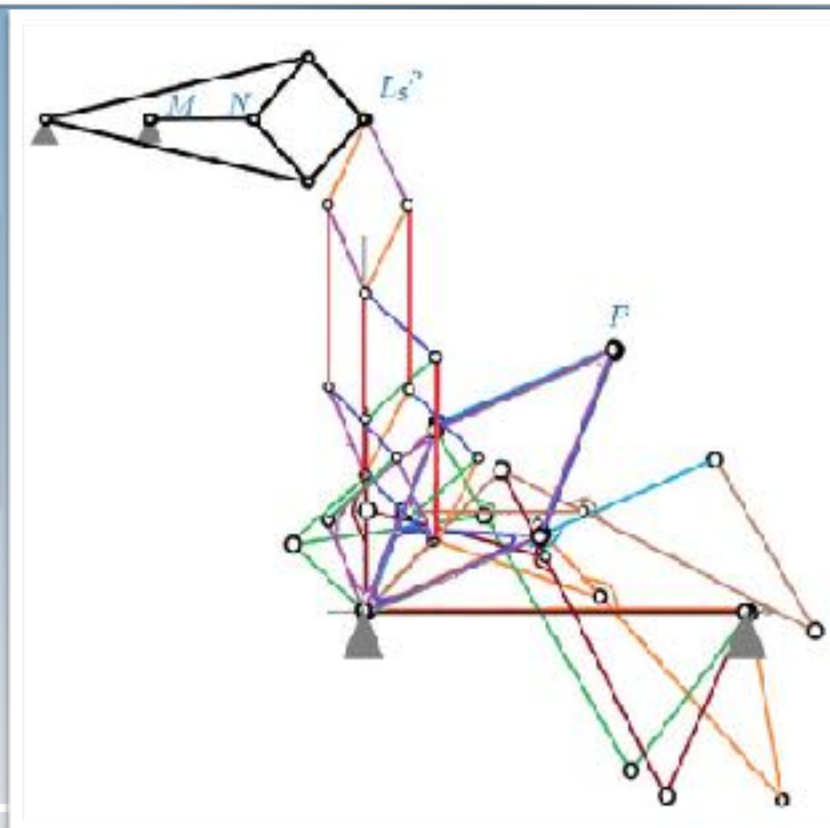
[b]Department of Mathematics, University of Maryland, College Park, MD 20742, USA

Received 7 September 2000; accepted 23 January 2001

Theorem 11.2. *Let $f = f(z,\bar{z})$, $f : \mathbb{C} \to \mathbb{R}$ be a polynomial function of the variables $z, \bar{z}$ and $\Gamma := f^{-1}(0) \subset \mathbb{C}$ be a real-algebraic curve. Pick an open (in the classical topology) bounded subset $U \subset \Gamma$. Then there is a closed $\mathbb{C}$-functional linkage $\mathcal{L}_0$ so that the input map $p_0 : C(\mathcal{L}_0, Z) \to \mathbb{C}$ is an analytically trivial polynomial covering over U.*

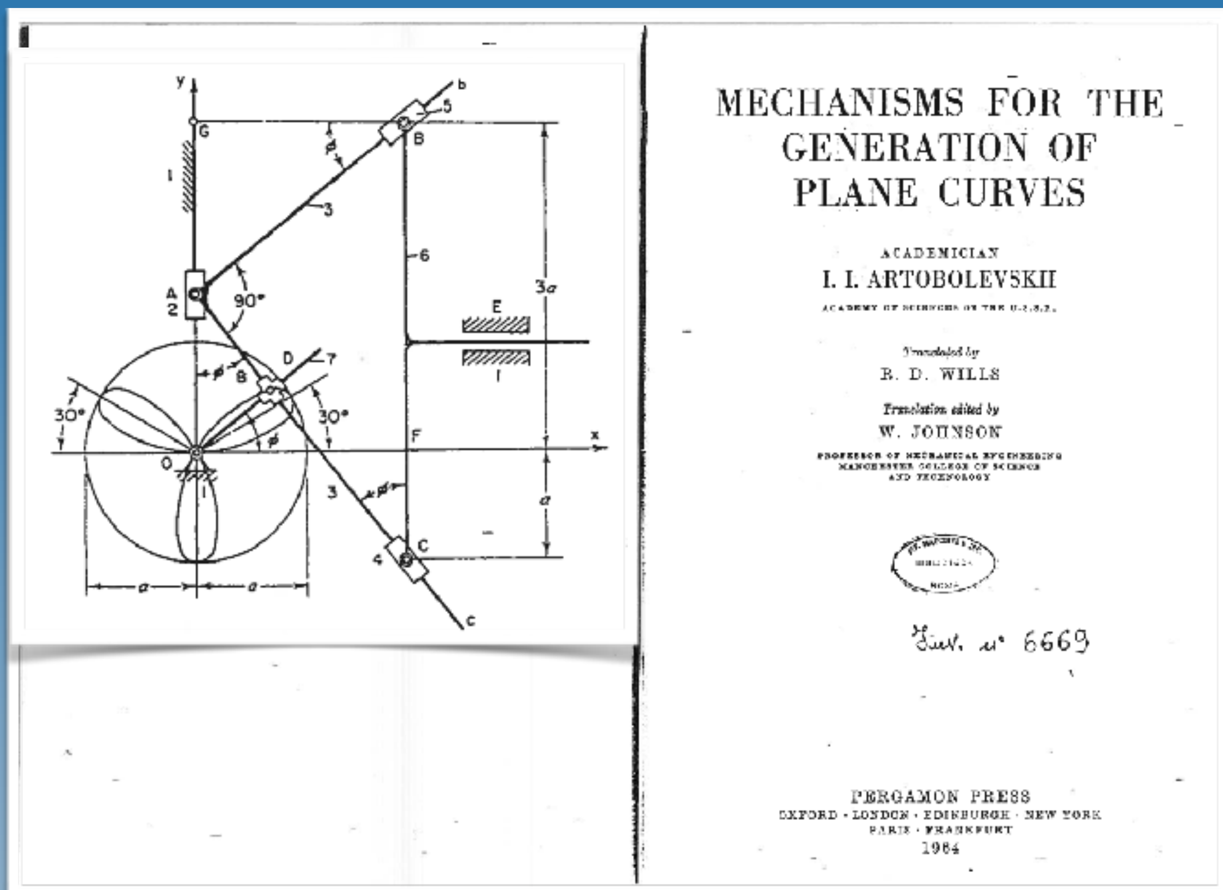
Saxena provides a useful demonstration of the design process to obtain a linkage consisting of 48 links and 70 joints to draw the curve:

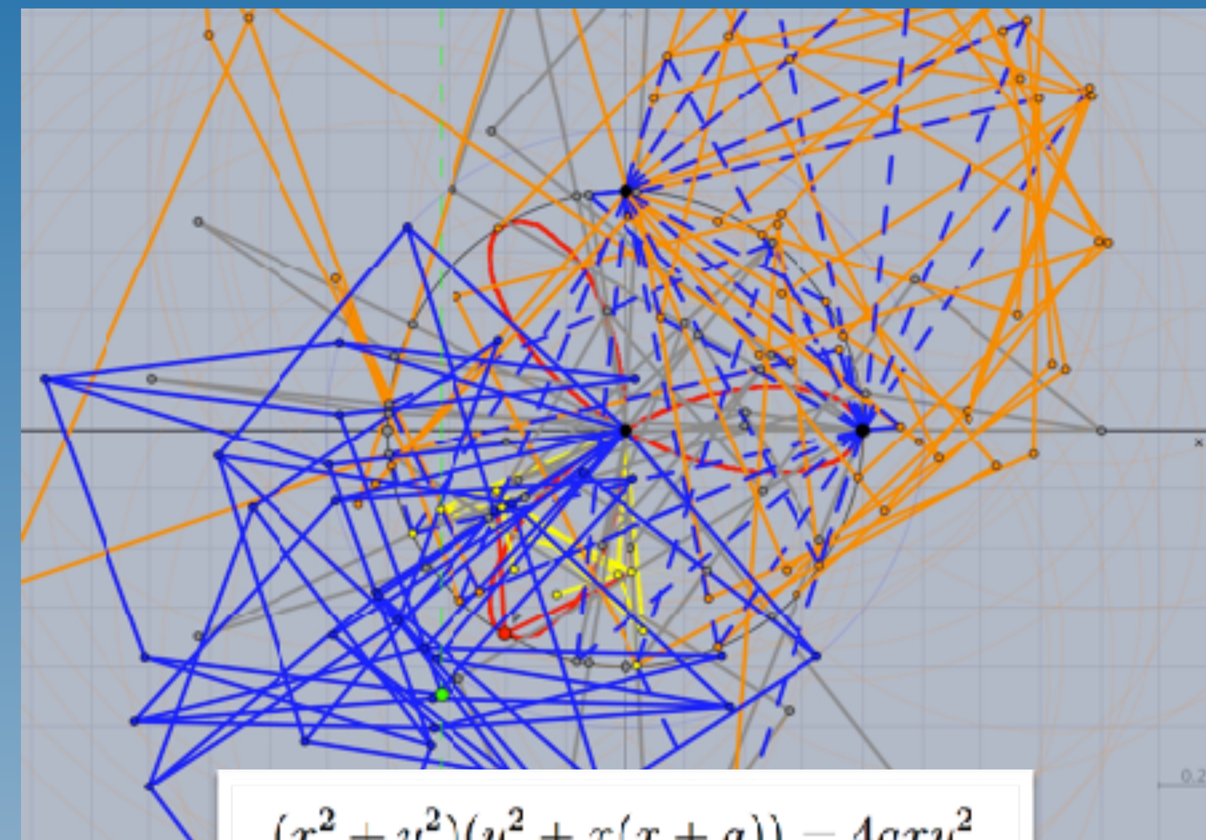
$$C = (x-y)(x+y+2a) = 0$$



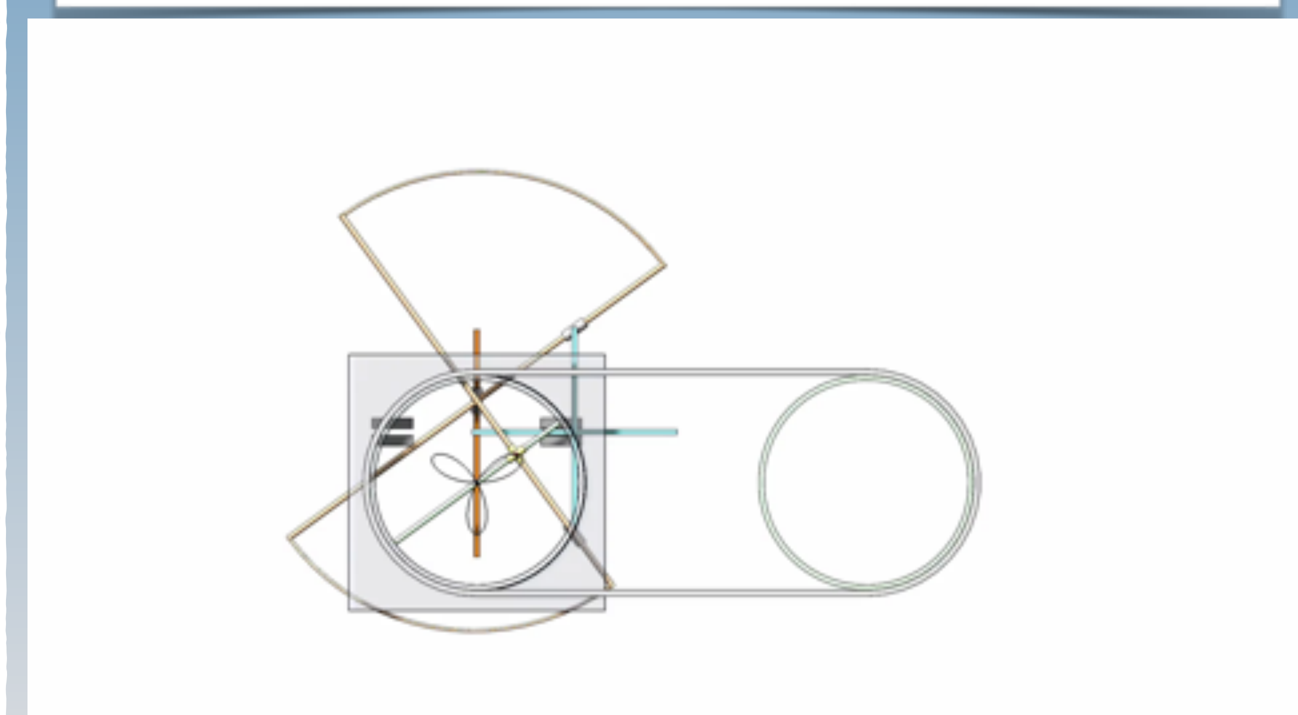
Saxena, A.: Kempe's linkages and the universality theorem. Resonance March, 220–237 (2011)

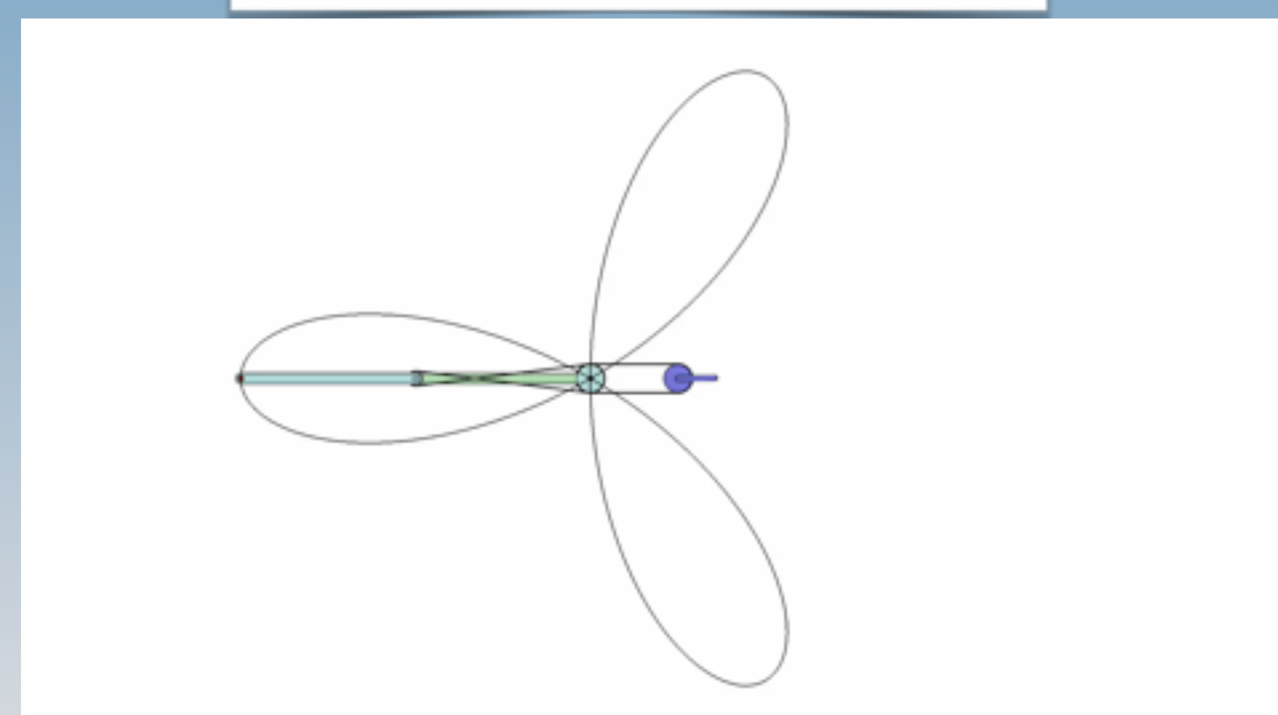
The Trifolium Curve





$$(x^2 + y^2)(y^2 + x(x + a)) = 4axy^2.$$

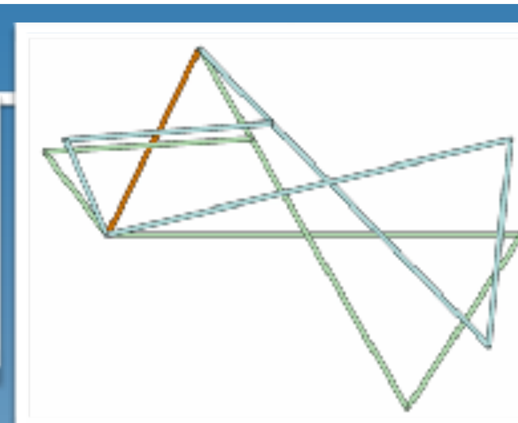


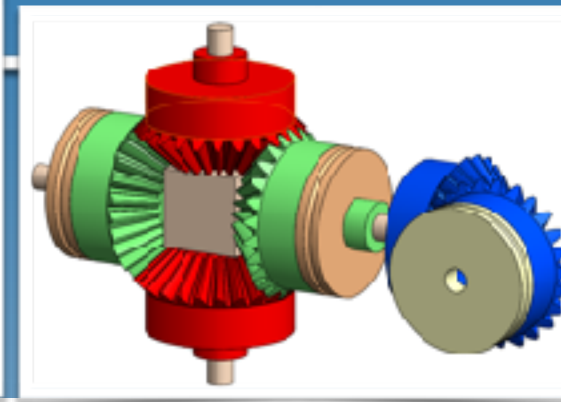


Mechanical Computation

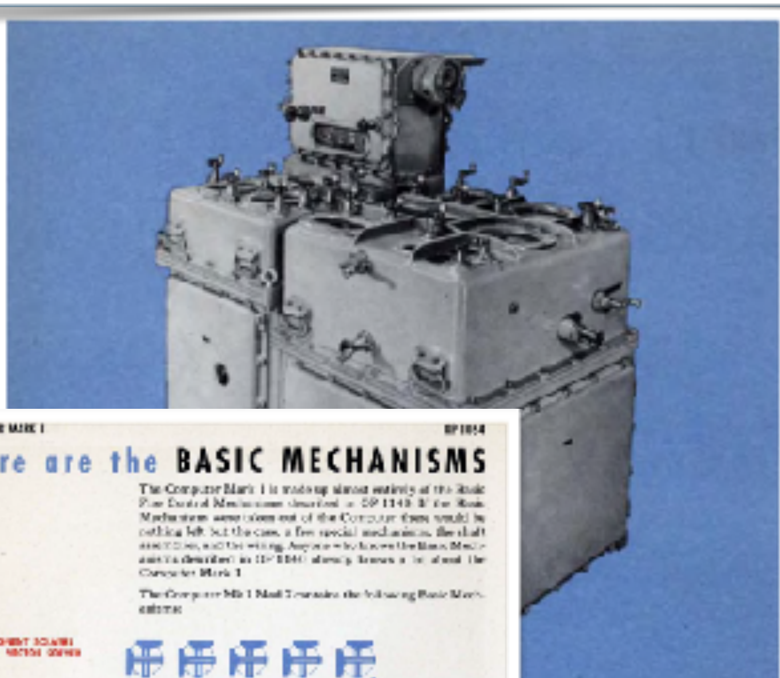
Replace Kempe's linkages for addition, negation, multiplication and translation with the equivalent elements:

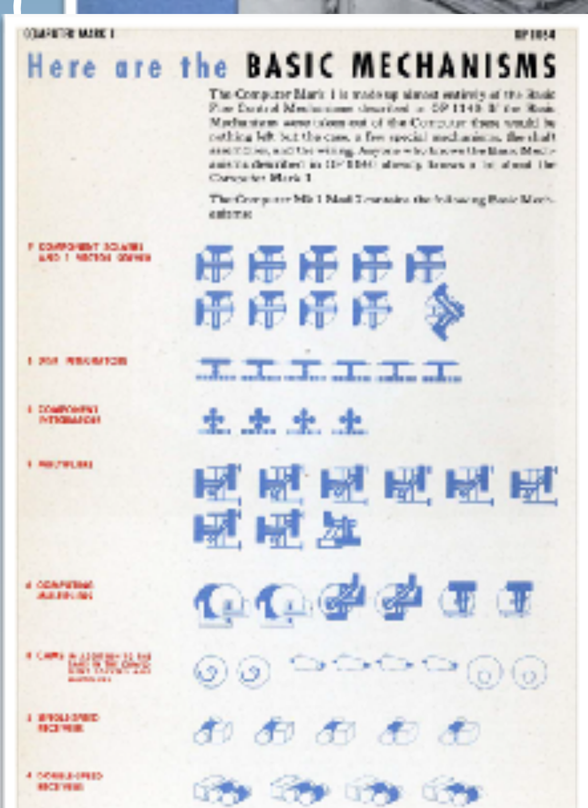
Use mechanical computation:
a bevel gear differential to add angles, and
cable drives to reverse, multiply and translate

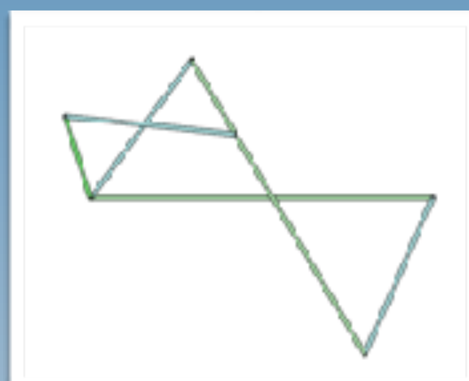


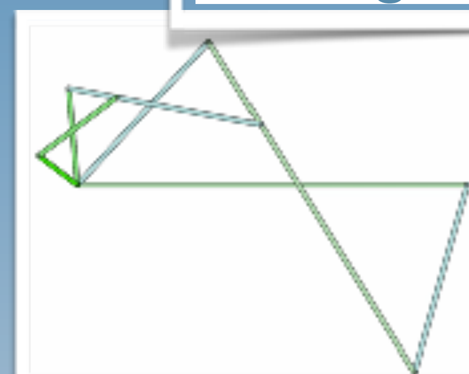


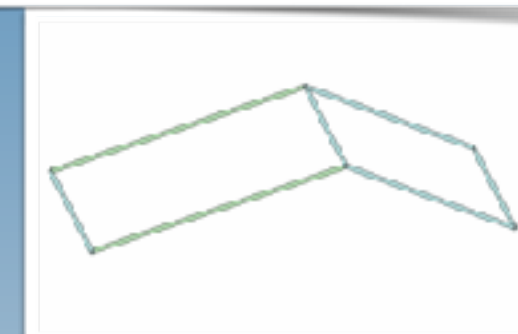
Bevel gear differential to add.

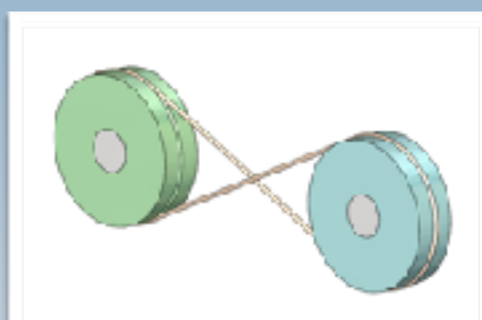


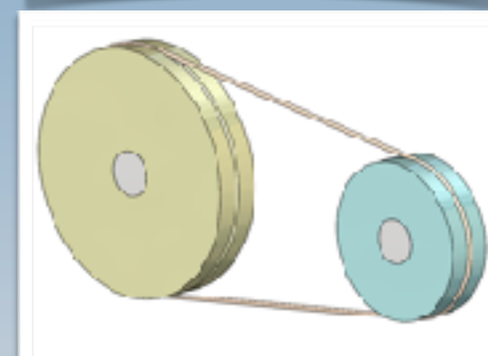


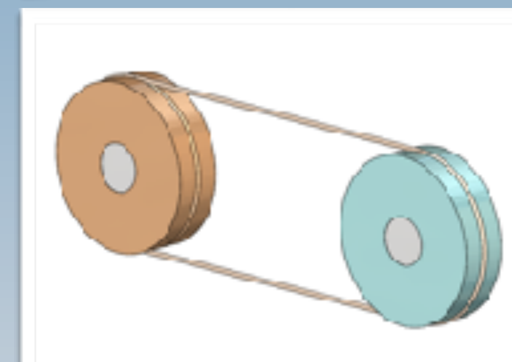












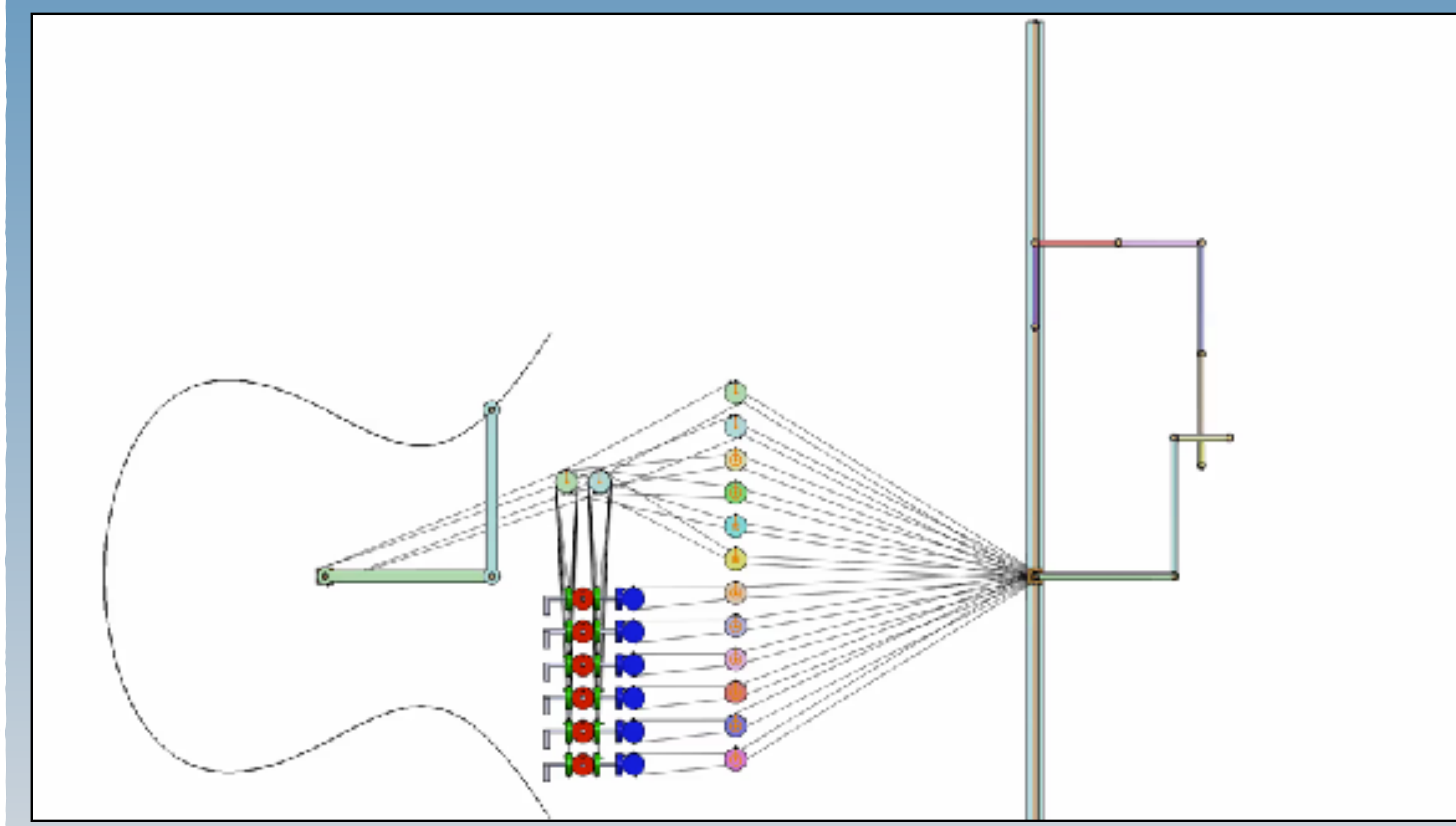
Cable drives to reverse, multiply and translate.

Drawing Algebraic Curves

$$f(x,y) = x^3 - y^2 - x + 1 = 0$$

$$\mathbf{P} = \begin{Bmatrix} x(\theta,\phi) \\ y(\theta,\phi) \end{Bmatrix} = \begin{Bmatrix} L_1\cos\theta + L_2\cos\phi \\ L_1\sin\theta + L_2\sin\phi \end{Bmatrix}$$

$$f(\theta,\phi) = \frac{5}{4}\cos\theta + \frac{5}{4}\cos\phi + \frac{1}{2}\cos 2\theta + \frac{1}{2}\cos 2\phi + \frac{1}{4}\cos 3\theta + \frac{1}{4}\cos 3\phi + \cos(\theta - \phi + \pi)$$
$$+\cos(\theta+\phi) + \frac{3}{4}\cos(\theta - 2\phi) + \frac{3}{4}\cos(\theta + 2\phi) + \frac{3}{4}\cos(2\theta - \phi) + \frac{3}{4}\cos(2\theta + \phi) = 0$$

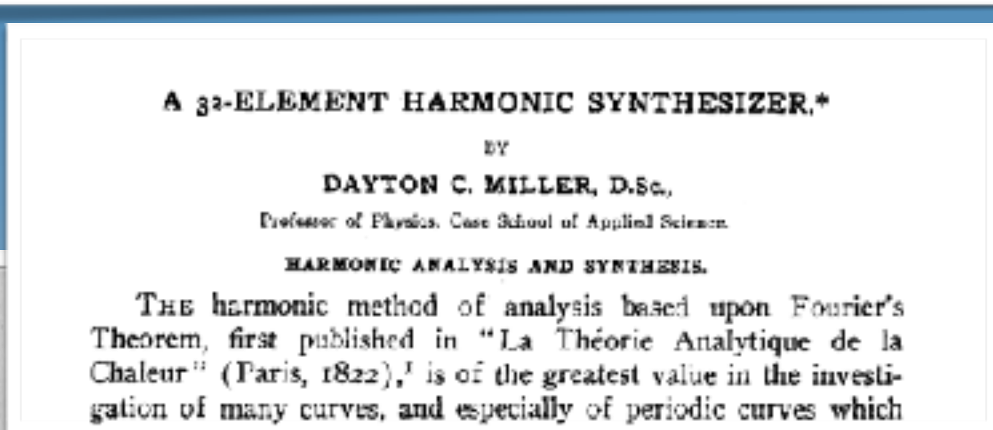


Alex Kobel (http://www.a-kobel.de/kempe/)

Link Number	Link Length	Phase	Angular Velocity
L_1	1.25	0	θ
L_2	1.25	90	ϕ
L_3	0.5	0	2θ
L_4	0.5	180	2ϕ
L_5	0.25	0	3θ
L_6	0.25	270	3ϕ
L_7	1	90	θ-ϕ
L_8	1	90	θ+ϕ
L_9	0.75	-180	θ-2ϕ
L_{10}	0.75	180	θ+2ϕ
L_{11}	0.75	-90	2θ-ϕ
L_{12}	0.75	90	2θ+ϕ

Mechanical Fourier Series

1. A closed parameterized curve, f(t)=(x(t), y(t)), has a Fourier series expansion of the coordinate functions, x(t) and y(t);
2. Scotch yoke mechanisms generate cosine and sine terms, add using a belt and pulleys;
3. Combine the movements of x and y coordinates to draw the curve.



A 32-ELEMENT HARMONIC SYNTHESIZER.*

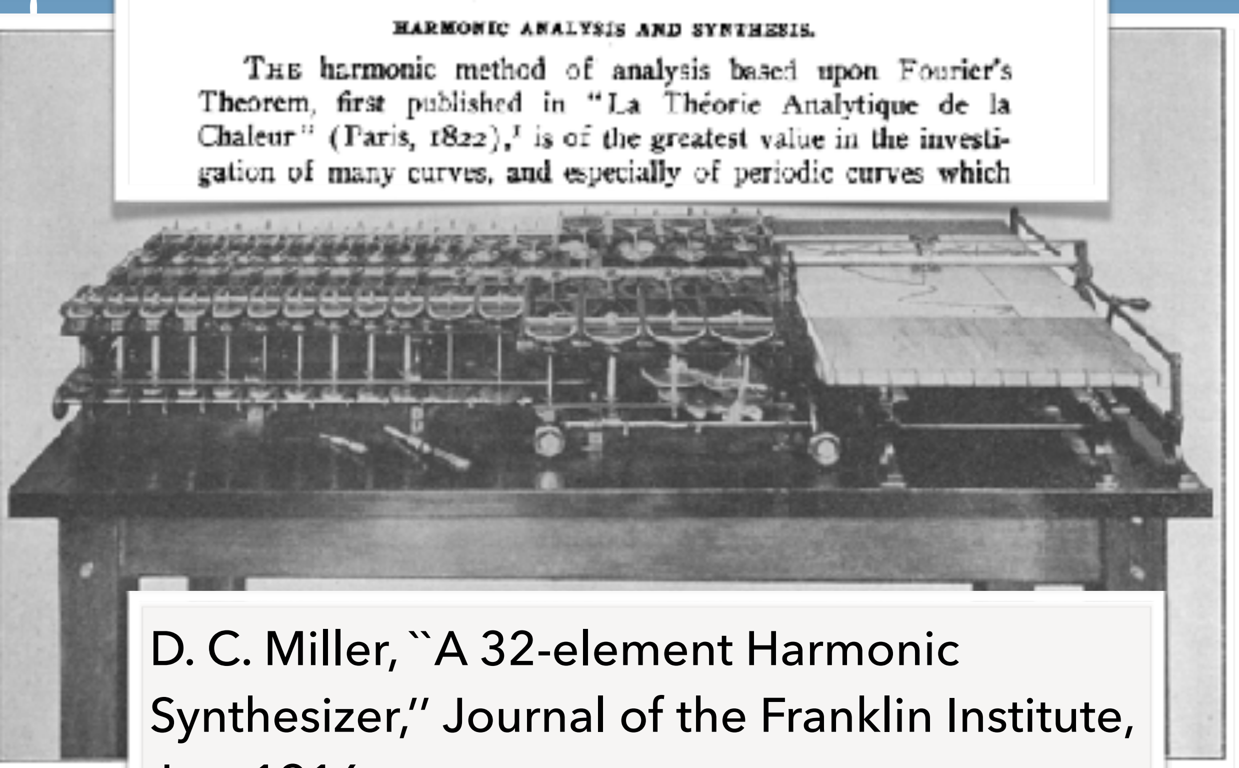
BY

DAYTON C. MILLER, D.Sc.,

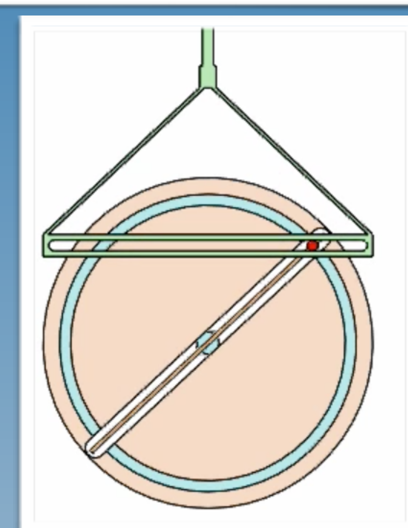
Professor of Physics, Case School of Applied Science.

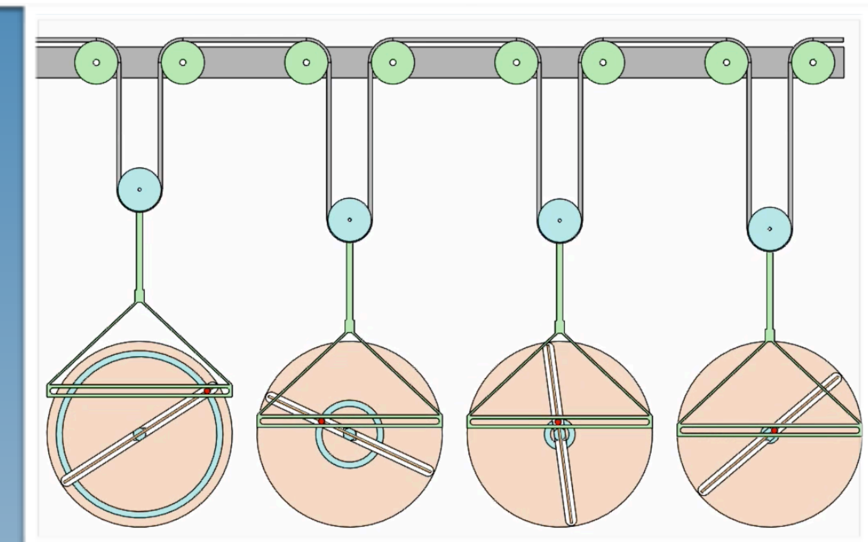
HARMONIC ANALYSIS AND SYNTHESIS.

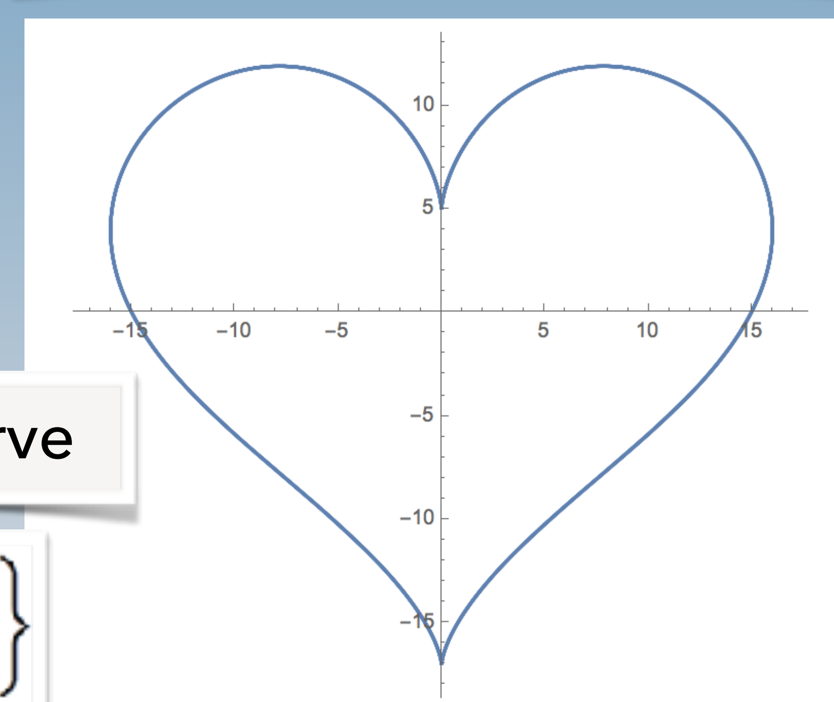
The harmonic method of analysis based upon Fourier's Theorem, first published in "La Théorie Analytique de la Chaleur" (Paris, 1822),[1] is of the greatest value in the investigation of many curves, and especially of periodic curves which



D. C. Miller, ``A 32-element Harmonic Synthesizer,'' Journal of the Franklin Institute, Jan. 1916.





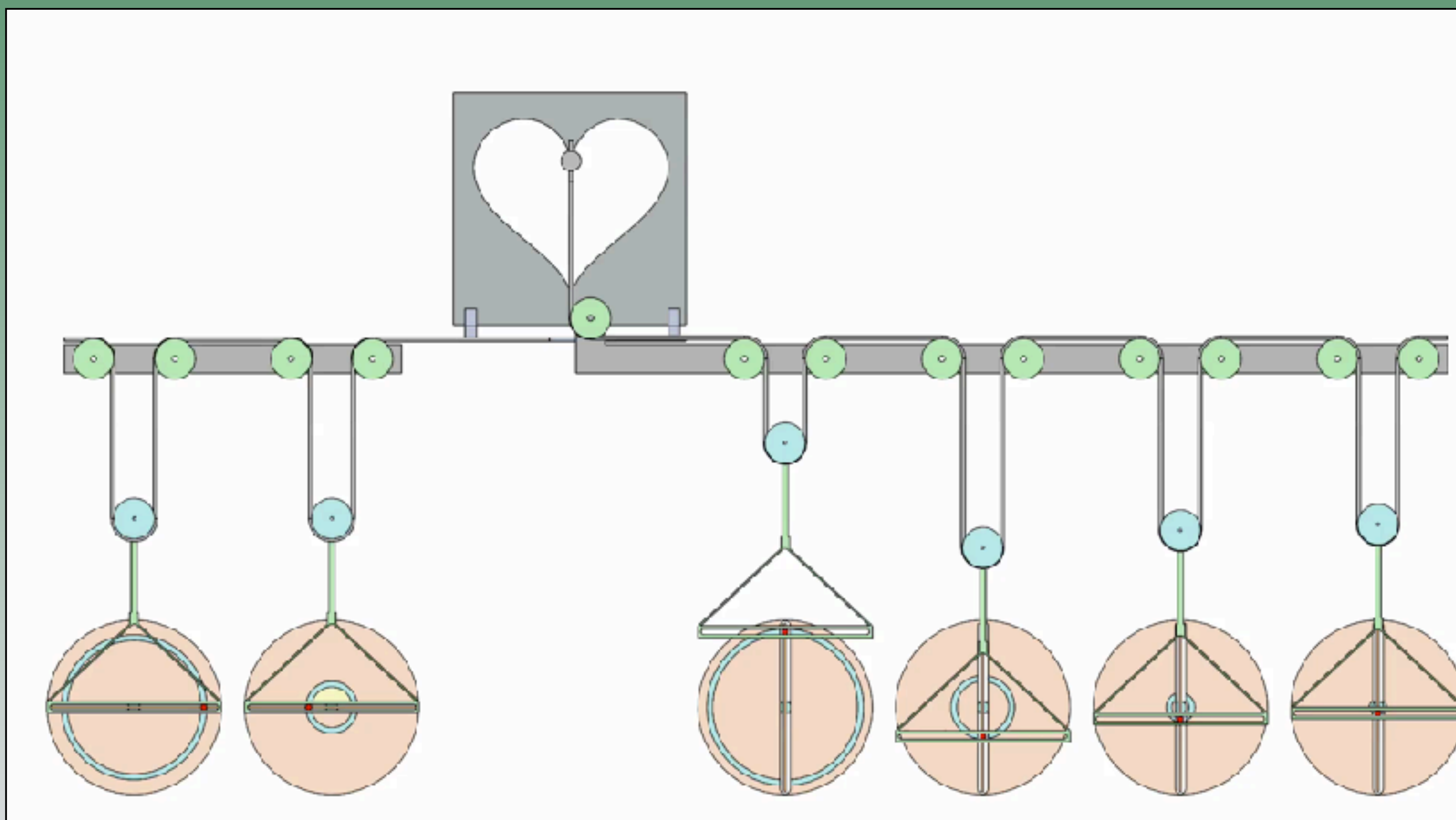


The Heart Curve

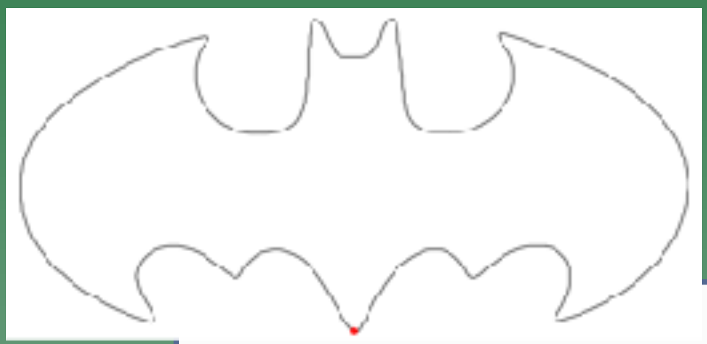
$$\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} = \begin{Bmatrix} 12\sin t + 4\sin(-\pi + 3t) \\ 13\sin(\frac{\pi}{2} + t) + 5\sin(\frac{3\pi}{2} + 2t) + 2\sin(\frac{3\pi}{2} + 3t) + \sin(\frac{3\pi}{2} + 4t) \end{Bmatrix}$$

Trigonometric Plane Curves

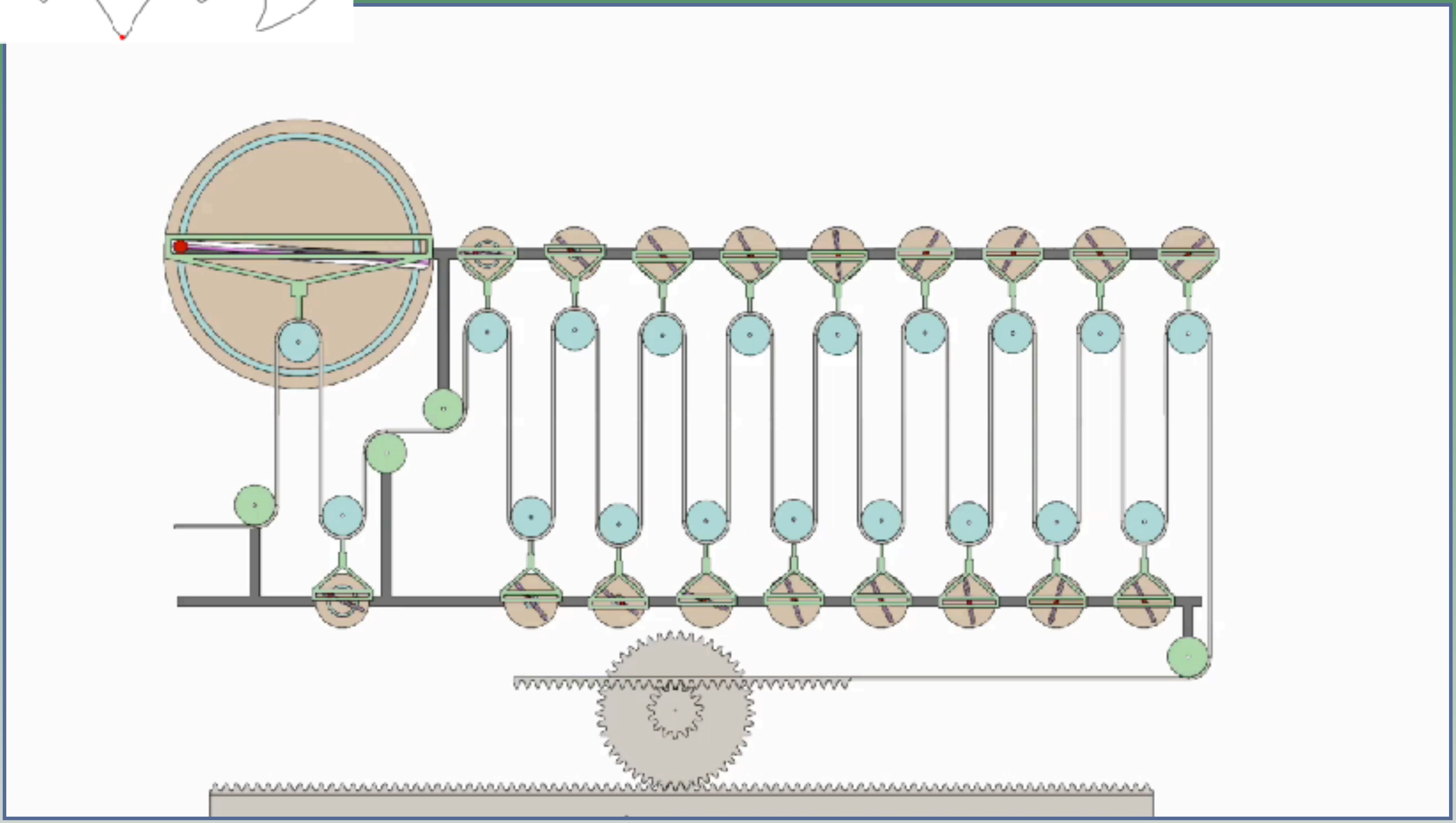
$$\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} = \begin{Bmatrix} 12\sin t + 4\sin(-\pi + 3t) \\ 13\sin(\frac{\pi}{2} + t) + 5\sin(\frac{3\pi}{2} + 2t) + 2\sin(\frac{3\pi}{2} + 3t) + \sin(\frac{3\pi}{2} + 4t) \end{Bmatrix} = \begin{Bmatrix} \sum_{k=0}^{m} a_k \cos k\theta + b_k \sin k\theta \\ \sum_{k=0}^{m} c_k \cos k\theta + d_k \sin k\theta \end{Bmatrix} \quad (1)$$



Curves Defined by Points



The Discrete Fourier Transform of boundary points yields a trigonometric curve z(t)=(x(t), y(t)).

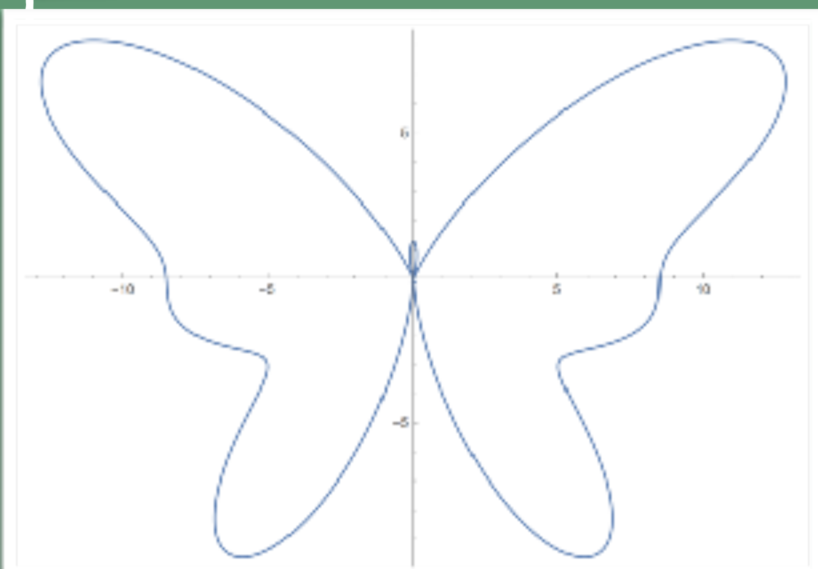


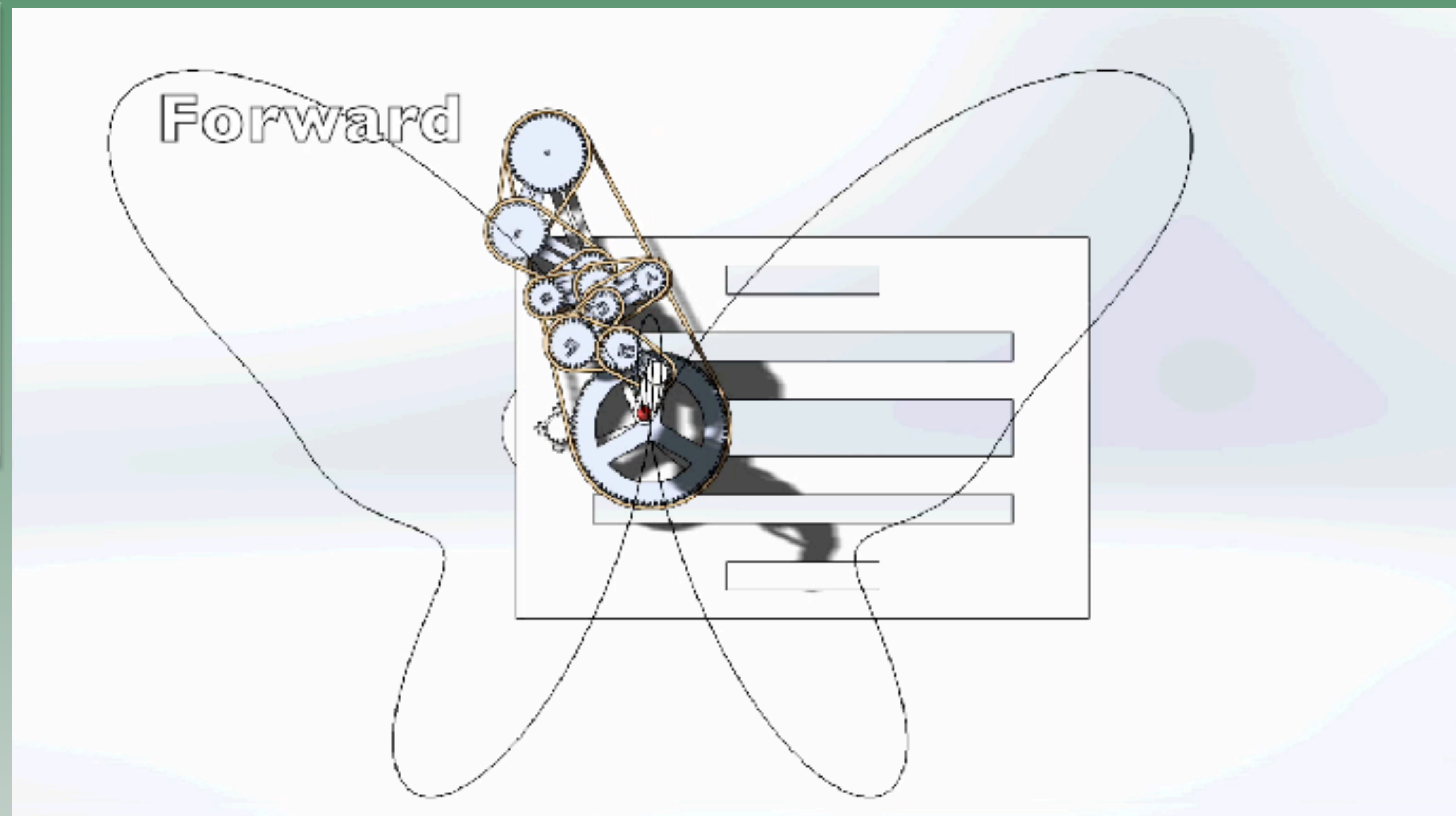
The logo boundary of 3235 points yielded 19 terms for x(t) and y(t).

Epicycles: The Coupled Serial Chain

The Butterfly mechanism draws the Butterfly curve:

$$\mathbf{z}(t) = \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} = \begin{cases} 9\cos(t) + 0.75\cos(3t) - 1.25\cos(5t) + 0.65\sin(2t) + 2.4\sin(4t) + 0.25\sin(6t) - 1.2\sin(8t) - 0.2\sin(10t) \\ -0.5 + 1.65\cos(2t) + 0.1\cos(4t) - 2.25\cos(6t) + 0.8\cos(8t) + 0.2\cos(10t) + 5\sin(t) + 3.25\sin(3t) - 1.25\sin(5t) \end{cases}$$





Y. Liu and J. M. McCarthy, "Design of Mechanisms to Draw Trigonometric Plane Curves," special issue "Selected Papers from the IDETC 2016," Journal of Mechanisms and Robotics, April 2017, Vol 9(2). doi: 10.1115/1.4035882

Manufacturing Prototypes

The Trigonometric Bezier Curve

A cubic Bezier curve is defined by four control points.

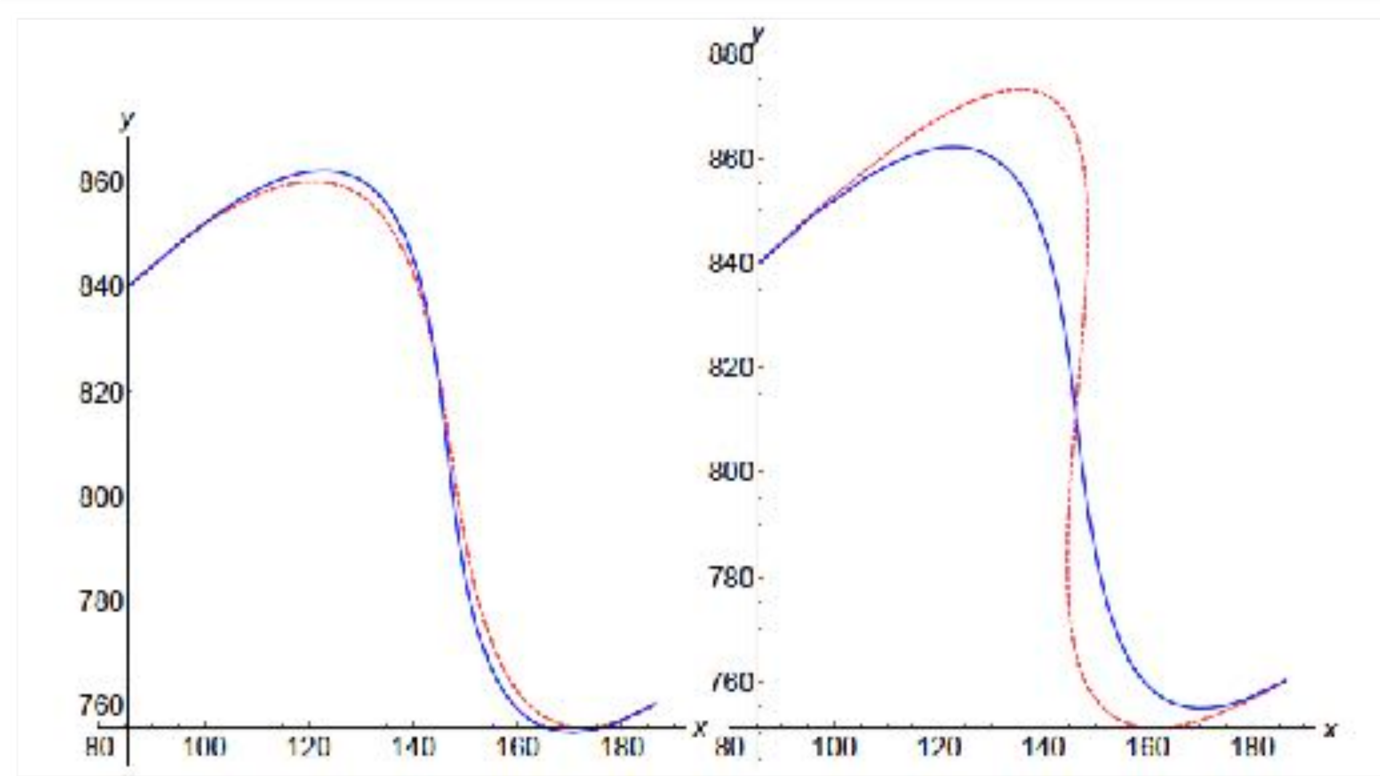
$$\mathbf{r}(t) = (1-t)^3\mathbf{P}_0 + 3(1-t)^2 t\mathbf{P}_1 + 3(1-t)t^2\mathbf{P}_2 + t^3\mathbf{P}_3, \quad 0 \leqslant t \leqslant 1.$$

A cubic Trigonometric Bezier curve can be defined by the same four control points.

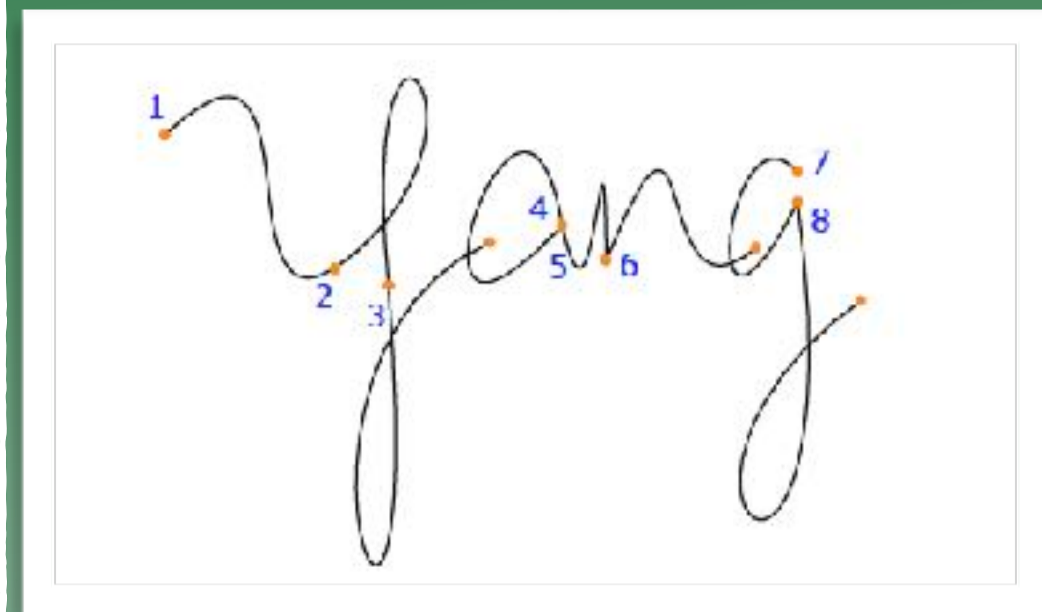
$$\begin{aligned} s(t,\lambda) &= (1-\sin\tfrac{\pi t}{2})^2(1-\lambda\sin\tfrac{\pi t}{2})\mathbf{P}_0 + \sin\tfrac{\pi t}{2}(1-\sin\tfrac{\pi t}{2})(2+\lambda(1-\sin\tfrac{\pi t}{2}))\mathbf{P}_1 \\ &+ \cos\tfrac{\pi t}{2}(1-\cos\tfrac{\pi t}{2})(2+\lambda(1-\cos\tfrac{\pi t}{2}))\mathbf{P}_2 + (1-\cos\tfrac{\pi t}{2})^2(1-\lambda\cos\tfrac{\pi t}{2})\mathbf{P}_3, \\ &\qquad 0 \leqslant t \leqslant 1. \end{aligned}$$

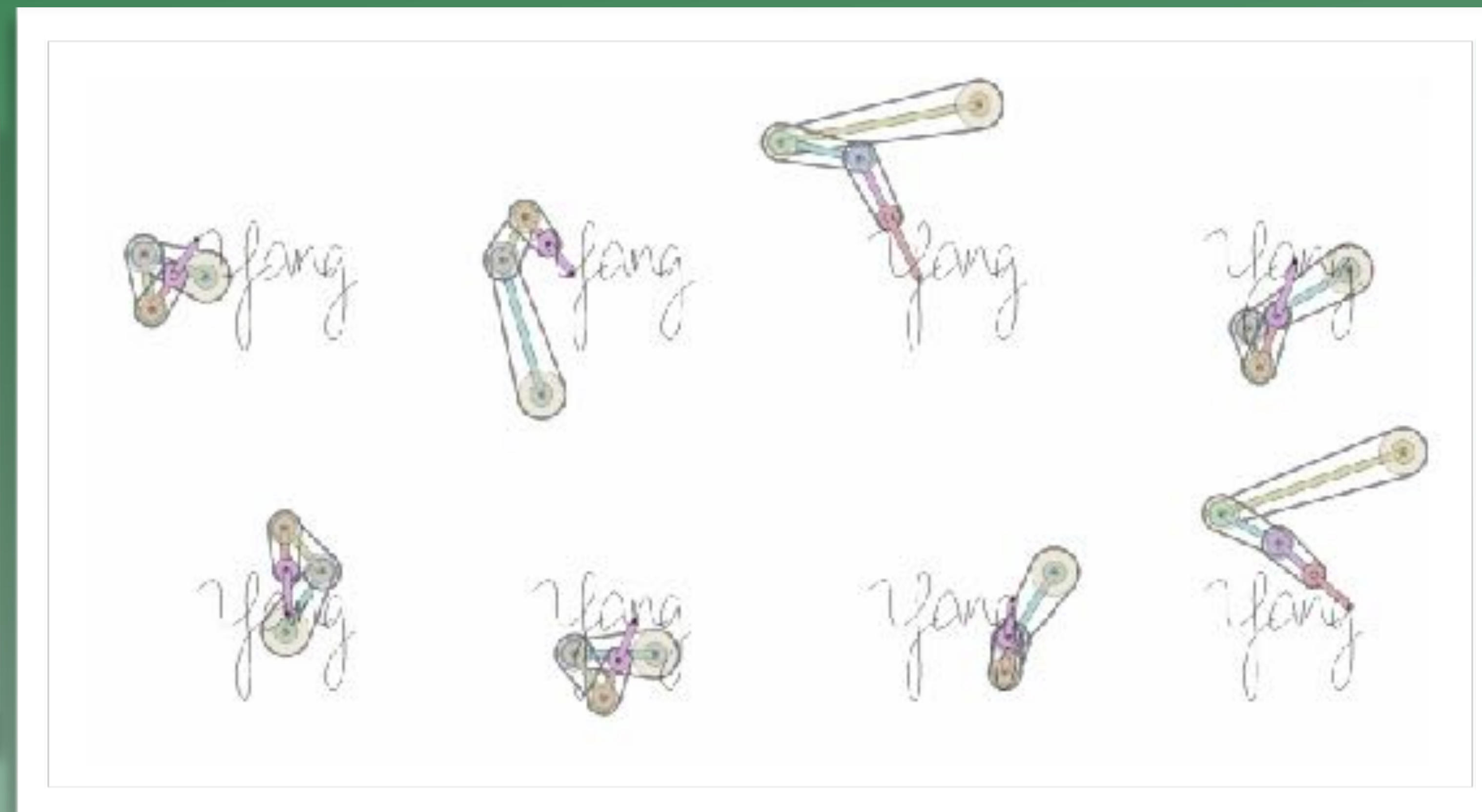
The shape parameter provides adjustment of the shape of the trigonometric Bezier curve to fit the original Bezier curve.

Y. Liu and J. M. McCarthy, "Design of a Linkage to Draw a Bezier Curve." Submitted to Mechanism and Machine Theory.



A Linkage Signs Your Name

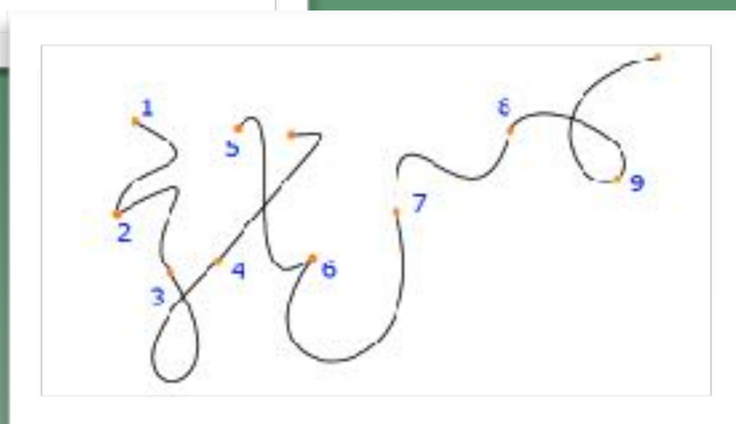


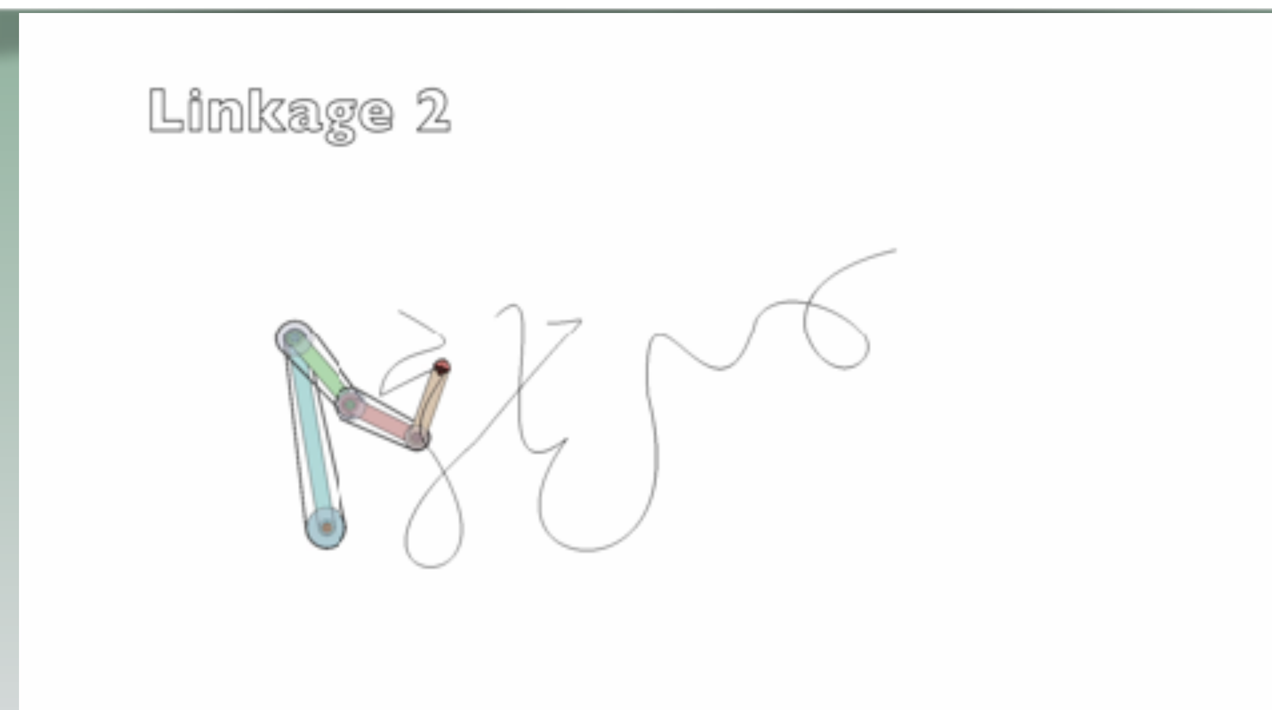


Each of the serial chains is driven by the same input.

The system has one degree-of-freedom

A Linkage Writes Chinese





Each of the serial chains is driven by the same input.

The system has one degree-of-freedom

Freudenstein's Path Synthesis

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Synthesis of Path-Generating Mechanisms by Numerical Methods[1]

Algebraic methods in kinematic synthesis are extended to cases in which the development of iterative numerical procedures are required. Algorithms are developed for the numerical solution of nonlinear, simultaneous, algebraic equations. Convergence is obtained without the need for a "good" initial approximation.

The theory is applied to the nine-point path synthesis of geared five-bar motion, in terms of which four-bar motion may be considered as a special case.

Introduction

The approximate synthesis of a given path by use of hinged mechanisms has been studied extensively in connection with four-bar mechanisms. Analytical [1][2] and graphical [2] solutions have been obtained for the problem specified in terms of five precision points and four crank angles; however, problems specified in terms of nine points (and no angles) have not been previously solved. Two published *formulations* of the nine-point path-synthesis problem are known to the authors [2, 3]. Both are for the four-bar mechanism; however, in the first no attempt is made to solve the equations, and in the second the suggested method of solution seems incomplete.

In this investigation we consider *geared five-bar mechanisms*, Fig. 1. Since they can generate a large variety of coupler curves [4, 5, 6], these linkages can be used for the solution of varied and complex design problems [7]. Their analysis is more involved than that of four-bar mechanisms, which can be considered as a special case of the geared five-bar—both mechanisms have equivalent coupler curves when the gear ratio is plus one [8, 9, 10, 11]. Previous geared five-bar path syntheses consist of a graphical-design procedure based on the two-degree-of-freedom property of the five-bar "loop" [12], and two analytical formulations of the prescribed crank-rotation problem [13, 19].

Four-bar linkages have (single) coupler links whose both hinge points describe a circular path. In contrast, five-bar linkages

parameters are eliminated at the start and the closure equations reduced to one (nonlinear) equation per precision condition [3]. Secondly, mathematical methods were developed in order to obtain convergence of the numerical iterations used in solving these equations. These mathematical methods, which are included in a digital computer program, contain the following new features:

1 The "bootstrap" procedure—this essentially eliminates the need for a "good" initial approximation.
2 The "position interchange" procedure—this reformulates the problem in order to eliminate the cause of nonconvergence.
3 The "quality-index-control" procedure—this assures convergence to solutions characterized by a reasonable ratio of maximum to minimum link length.

The Theory of Path Synthesis

Definition. Dimensional kinematic synthesis is the procedure of determining the dimensions of a mechanism from the required motion. When the synthesis is phrased in terms of generating a given curve, the procedure is called path synthesis.

Usually one does not attempt to generate the given curve exactly. In fact, only a limited class of motions could be so generated [18, 22], and in general it suffices if within a desired interval the generated curve is a good approximation to the given one. In this paper the approximate path-synthesis problem is formulated by specifying the location of the *precision points*

B. Roth and F. Freudenstein, **1963** "Synthesis of Path-Generating Mechanisms by Numerical Methods," ASME Journal of Engineering for Industry, 85:298-304, 1963.

IBM 7090 digital computer: $3m, 36bit, 32k core memory

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Complete Solution of the Nine-Point Path Synthesis Problem for Four-Bar Linkages

The problem of finding all four-bar linkages whose coupler curve passes through nine prescribed points has been a longstanding unsolved problem in kinematics. Using a combination of classical elimination, multihomogeneous variables, and numerical polynomial continuation, we show that there are generically 1442 nondegenerate solutions along with their Roberts cognates, for a total of 4326 distinct solutions. Moreover, a computer algorithm that computes all solutions for any given nine points has been developed.

Introduction

The approximate synthesis of a given path by use of four-bar linkages has been studied extensively. Formulations in terms of four or five precision points along with specifications on crank angles or the position of the hinges of the mechanism have been solved (Freudenstein and Sandor, 1959; Shigley and Uicker, 1980; Erdman and Sandor, 1984; Morgan and Wampler, 1989; Subbian and Flugrad, 1989). However, the problem of finding four-bar linkages whose coupler curve passes through nine precision points, which was formulated as early as 1923 (Alt), has until now defied complete solution. Since nine general precision points is the largest number that can be prescribed, this formulation gives a designer maximum control over the shape of the coupler curve.

of degenerate solutions. By the theory of "parameter polynomial continuation" (Morgan and Sommese, 1989), we may ignore all the degenerate solutions and use only the nondegenerate ones as start points in subsequent continuations to find all nondegenerate solutions to any other problem of the class. Thus, we have not only established the generic number of nondegenerate solutions to the problem, but also have developed an efficient computer algorithm for finding them.

In any particular example, not all of the 1442 × 3 = 4326 solutions are useful. Most give linkages with complex link lengths, whereas others give real linkages that exhibit branch or order defects, or that have poor transmission angles, etc.

C. W. Wampler, A. J. Sommese, and A. P. Morgan, **1992**. "Complete solution of the nine-point path synthesis problem for four-bar linkages," Journal of Mechanical Design, 114(1):153–159.

or order defects, it is often difficult to find an acceptable solution by trial-and-error procedures. Only by finding all [illegible]

The most concise formulation of the problem is obtained [illegible]

The entire computational cost of the numerical reduction was 331.9 hours of CPU time on an IBM 3081. (The IBM 3081 is about 1/3 as fast as an IBM 3090.) Fortunately, that is a one-time only expense, and subsequent solutions of the problem cost only a small fraction as much.

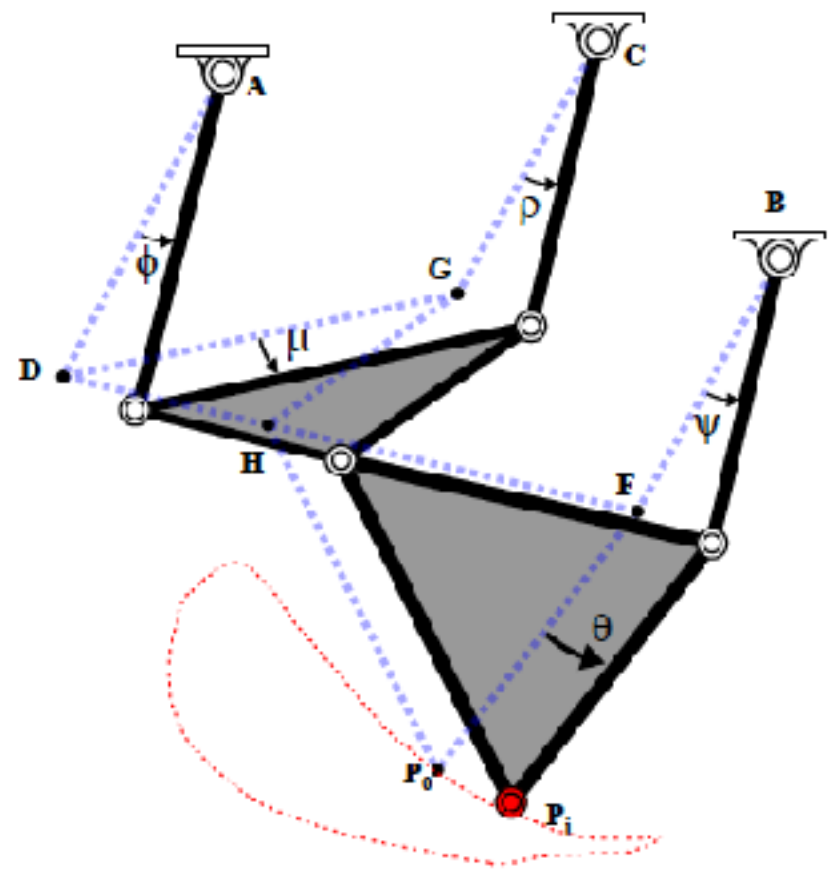
20 quadratic polynomials, total degree of $d=2^{20}=1{,}048{,}576$, multi-homogeneous degree $d=286{,}720$.
Today generic four-bar path synthesis is solved in minutes, and the parameter homotopy runs in seconds.

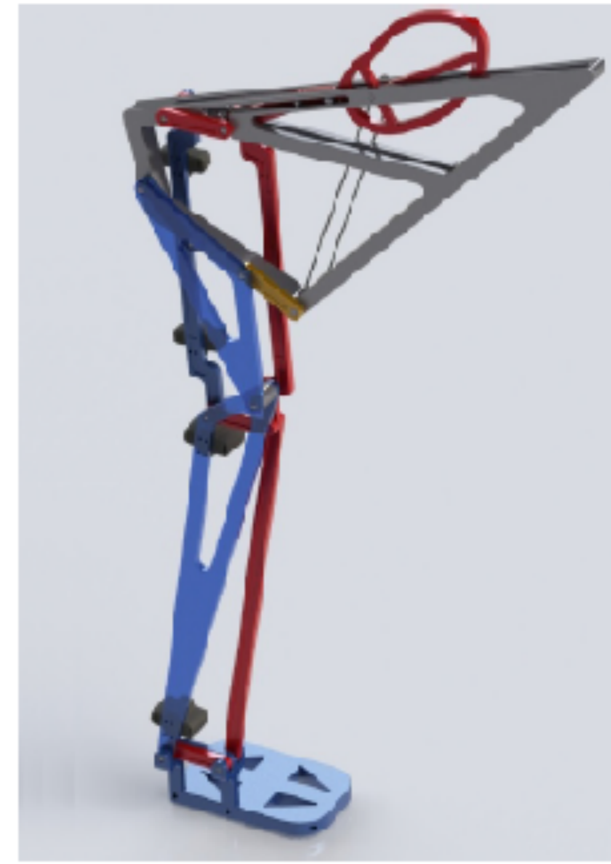
Unsolved: Six-bar Path Synthesis

N=15 points on a trajectory yields 154 quadratic equations in 154 unknowns.

Total degree $d=2^{154}$ or $d=2.3\times10^{46}$.

Beyond our current computation capabilities.





(b) Six-bar linkage guides a natural walking trajectory.

urve.

Normalization conditions and three pairs of loop equations:

$$\mathcal{A}_j: \begin{cases} Q_j(D-A)+U_j(H-D)+T_j(P_0-H)=P_j-A, \\ \bar{Q}_j(\bar{D}-\bar{A})+\bar{U}_j(\bar{H}-\bar{D})+\bar{T}_j(\bar{P}_0-\bar{H})=\bar{P}_j-\bar{A}, \end{cases} \quad j=1,\ldots,N-1,$$

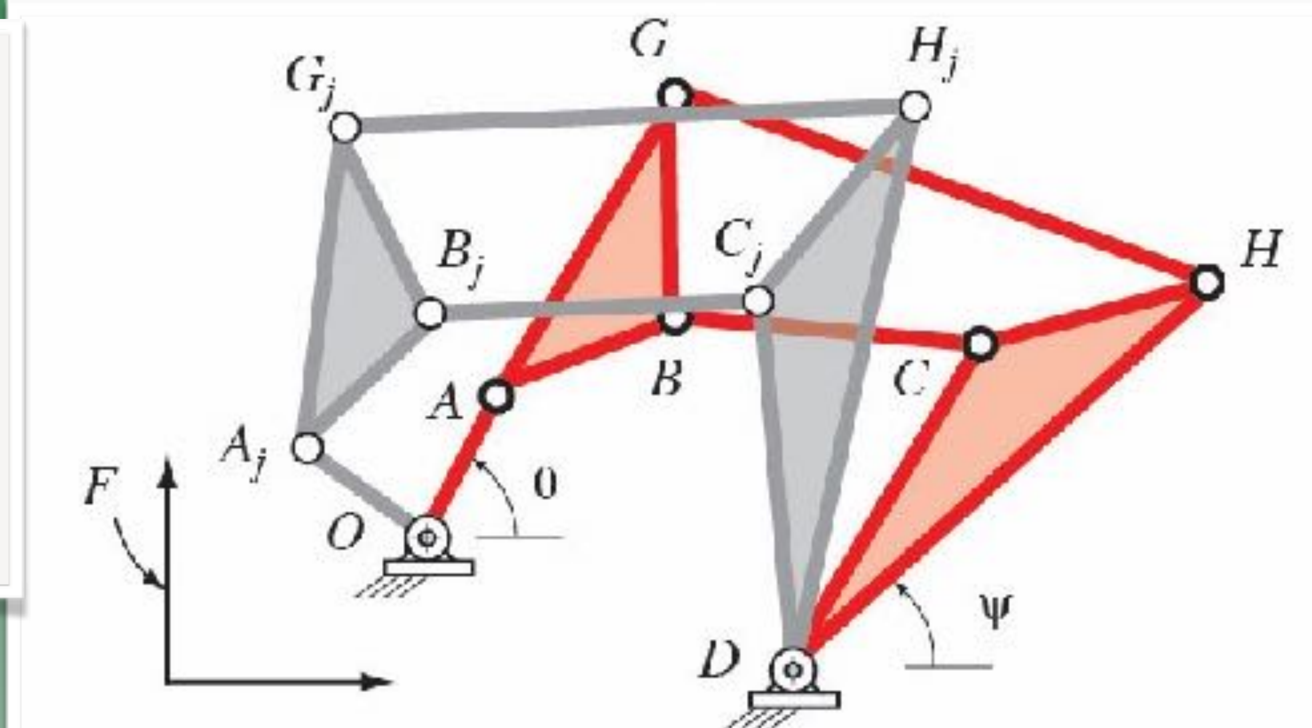
$$\mathcal{B}_j: \begin{cases} R_j(G-C)+U_j(H-G)+T_j(P_0-H)=P_j-C, \\ \bar{R}_j(\bar{G}-\bar{C})+\bar{U}_j(\bar{H}-\bar{G})+\bar{T}_j(\bar{P}_0-\bar{H})=\bar{P}_j-\bar{C}, \end{cases} \quad j=1,\ldots,N-1,$$

$$\mathcal{C}_j: \begin{cases} S_j(F-B)+T_j(P_0-F)=P_j-B, \\ \bar{S}_j(\bar{F}-\bar{B})+\bar{T}_j(\bar{P}_0-\bar{F})=\bar{P}_j-\bar{B}, \end{cases} \quad j=1,\ldots,N-1.$$

Solved: Six-bar Function Generation

For N=11 coordinated angles this yields 70 equations in 70 unknowns.
Bezout bound $d = 2^{70} = 1.18 \times 10^{21}$, the *multi-homogeneous degree is $d = 264 \times 10^{6}$.*

300 hrs on 256 nodes of a high performance computing cluster.



Three sets of normalization equations,

$$R_j\bar{R}_j = 1, \quad S_j\bar{S}_j = 1, \quad T_j\bar{T}_j = 1, \quad j = 1, \ldots, N-1.$$

Two sets of loop equations:

$$\mathcal{A}_j : \begin{cases} (D + S_j(C - D)) - (O + Q_j(A - O) + R_j(B - A)) = T_j(C - B), \\ (\bar{D} + \bar{S}_j(\bar{C} - \bar{D})) - (\bar{O} + \bar{Q}_j(\bar{A} - \bar{O}) + \bar{R}_j(\bar{B} - \bar{A})) - \bar{T}_j(\bar{C} - \bar{B}), \end{cases} \quad j = 1, \ldots, N-1.$$

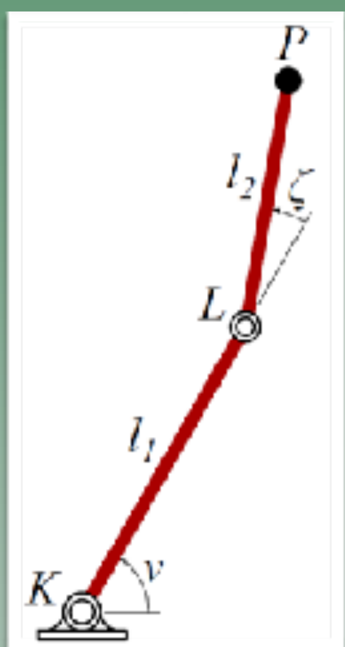
$$\mathcal{B}_j : \begin{cases} (D + S_j(H - D)) - (O + Q_j(A - O) + R_j((G - A)) = U_j(H - G), \\ (\bar{D} + \bar{S}_j(\bar{H} - \bar{D})) - (\bar{O} + \bar{Q}_j(\bar{A} - \bar{O}) + \bar{R}_j(\bar{G} - \bar{A})) = \bar{U}_j(\bar{H} - \bar{G}), \end{cases} \quad j = 1, \ldots, N \quad 1.$$

M. Plecnik and J. M. McCarthy, "Computational Design of Stephenson II Six-bar Function Generators for 11 Accuracy Points," ASME Journal of Mechanisms and Robotics, Vol 8(1), February 2016.

Modified Six-bar Path Synthesis

Path synthesis can be transformed to synthesis of a function generator

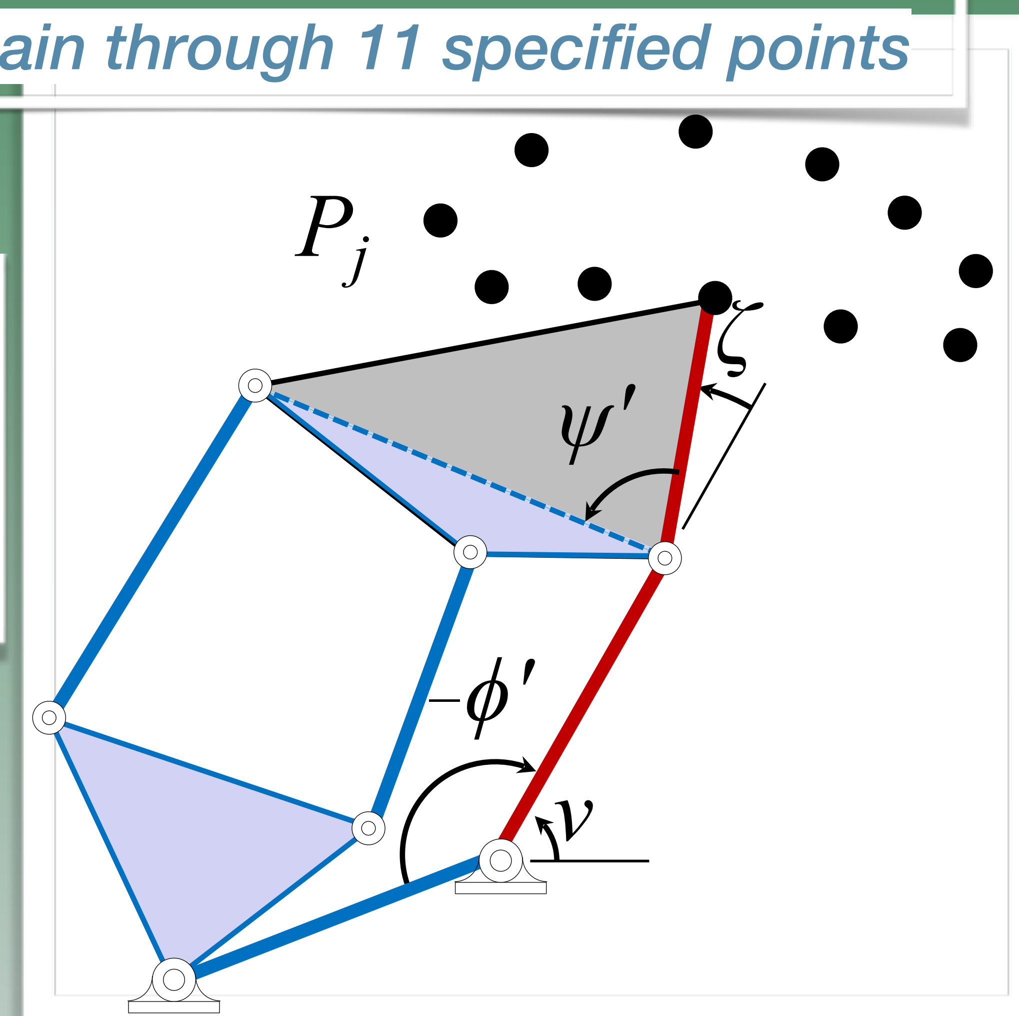
1. Specify an RR chain



2. Move the RR chain through 11 specified points

3. Compute the Inverse Kinematics of the RR chain to obtain the joint angle function

$(\nu_j, \zeta_j), j=0,\ldots,10$



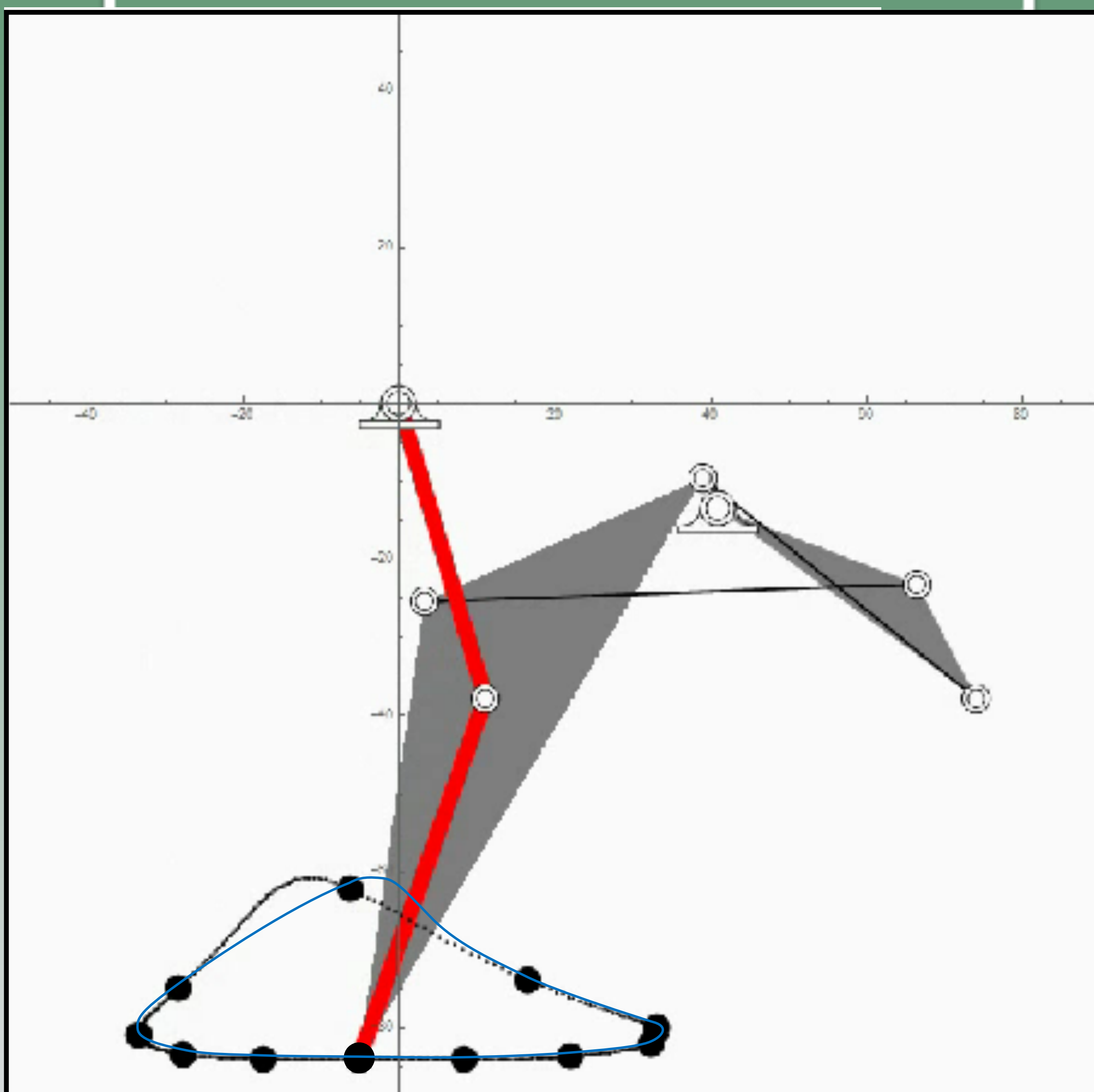
4. Solve the synthesis equations for 11 point Stephenson II function generators

5. Attach the function generator to the RR chain

Walking Machines

Theo Jansen designed an eight-bar linkage for the legs of his Strandbeest

We can design a six-bar linkage with a similar walking gait

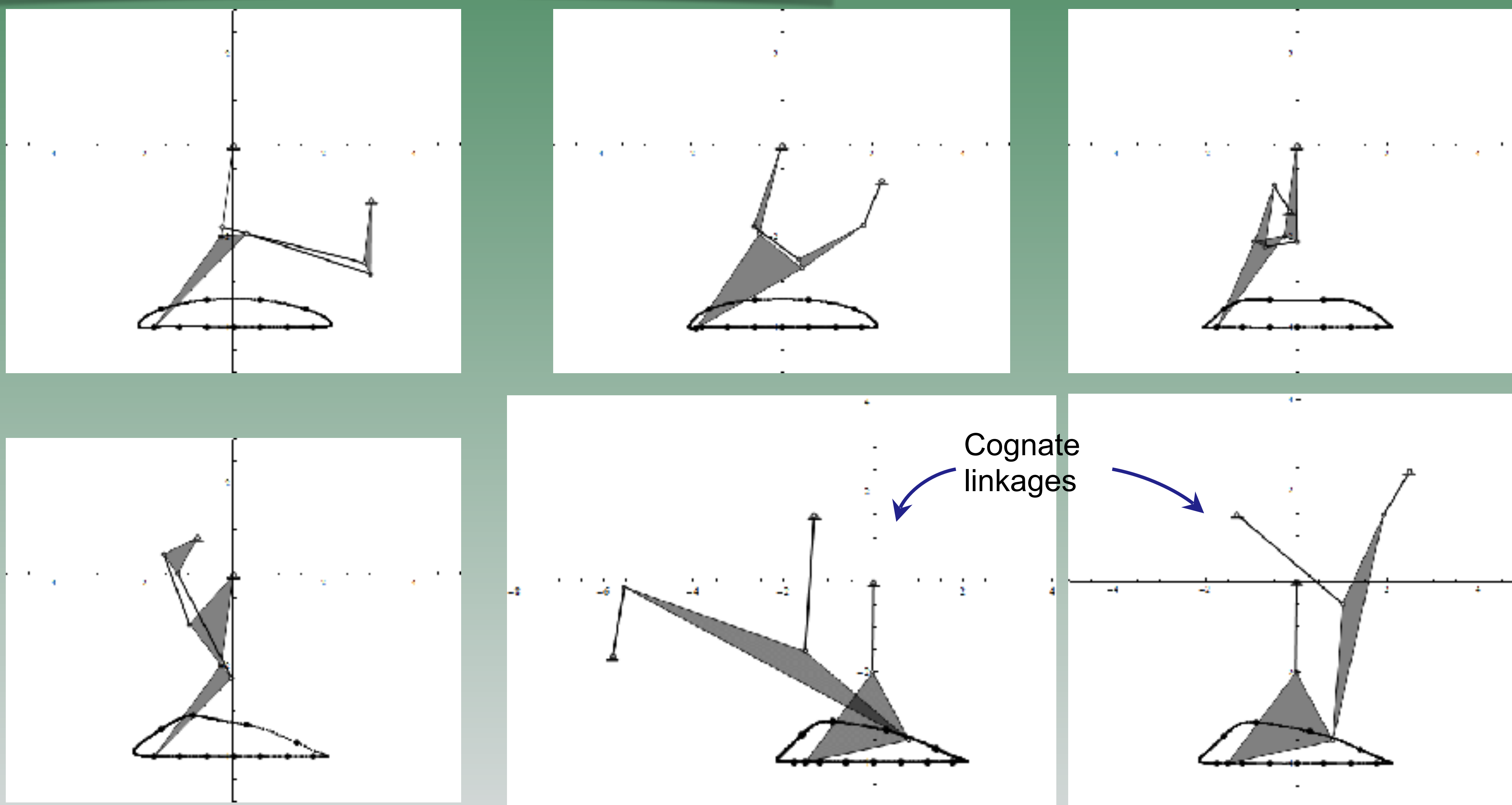


j	x	y
0	−5.160	−83.957
1	8.346	−84.026
2	21.993	−83.632
3	32.259	−82.128
4	33.018	−79.911
5	16.497	−73.889
6	−6.363	−62.120
7	−28.276	−74.865
8	−33.406	−80.964
9	−27.733	−83.440
10	−17.440	−84.032

Different Gaits

Once the general homotopy is solved, parameter homotopies execute rapidly.

Here are results for other foot trajectories

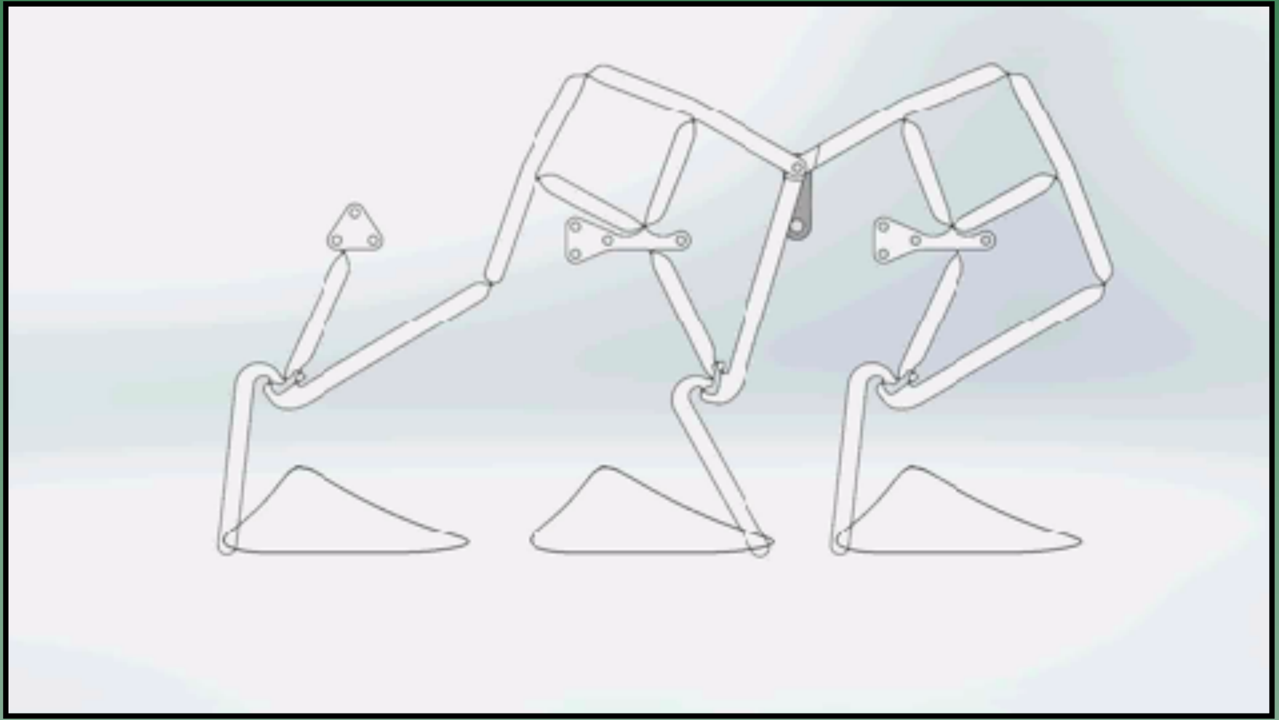


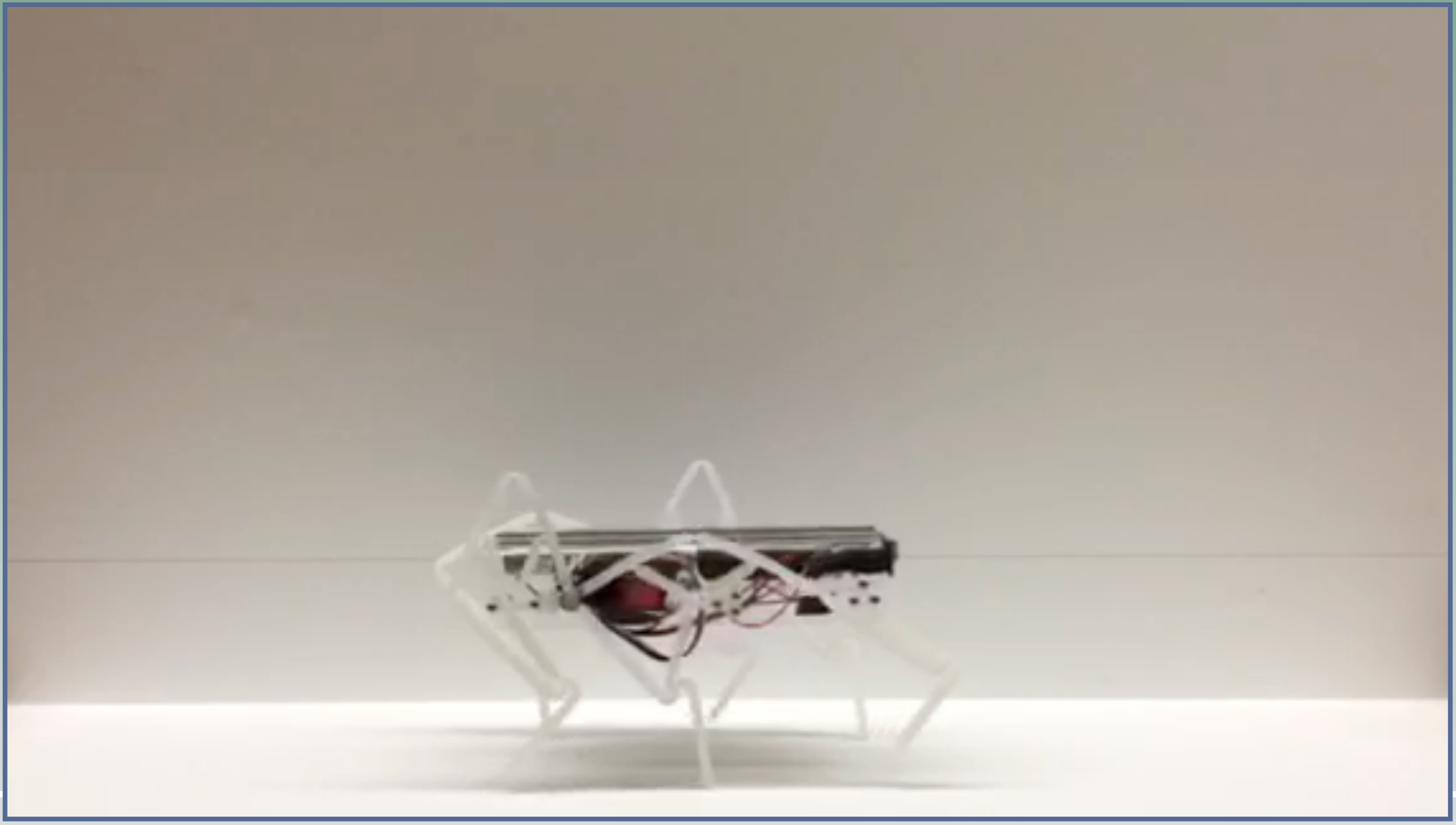
Prototype Walker

One of the leg designs was manufactured as a compliant linkage

Lasercut polypropylene

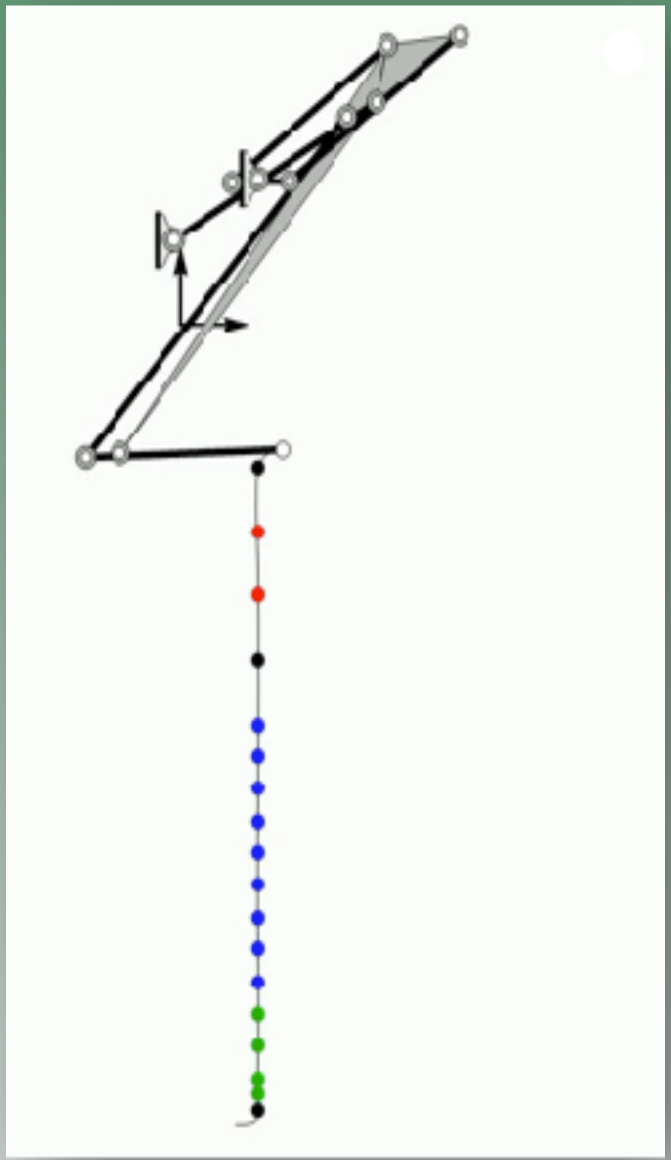
Used to build a robot about 30 cm in length



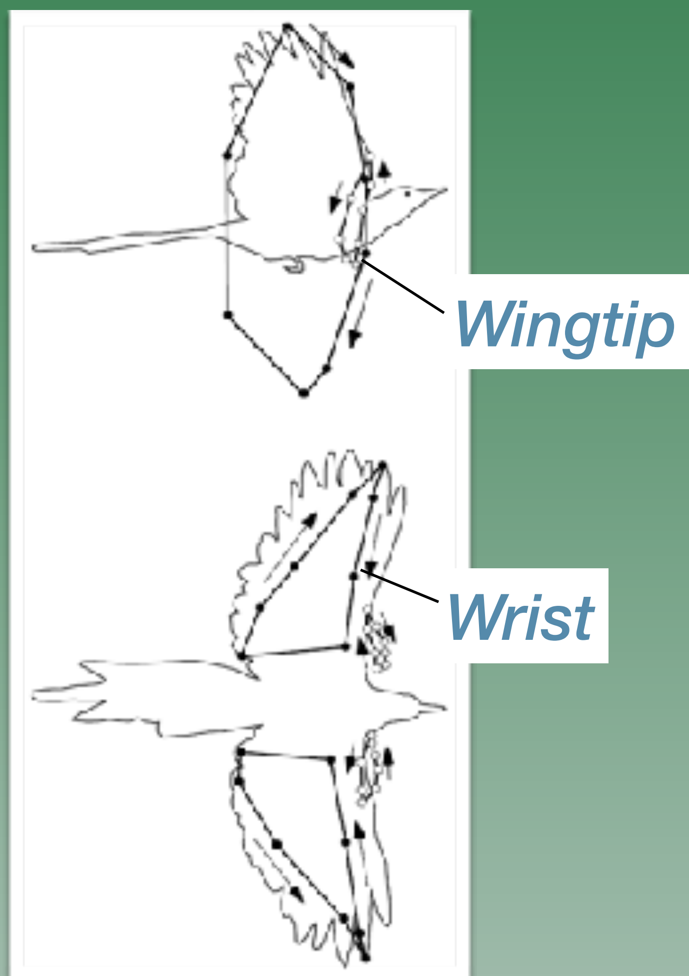


A Different Direction

D. W. Haldane, M. Plecnik, J. K. Yim, and R. S. Fearing, 2016. "Robotic Vertical Jumping Agility via Series-Elastic Power Modulation," Science Robotics, 1(1):



Wings Instead of Legs



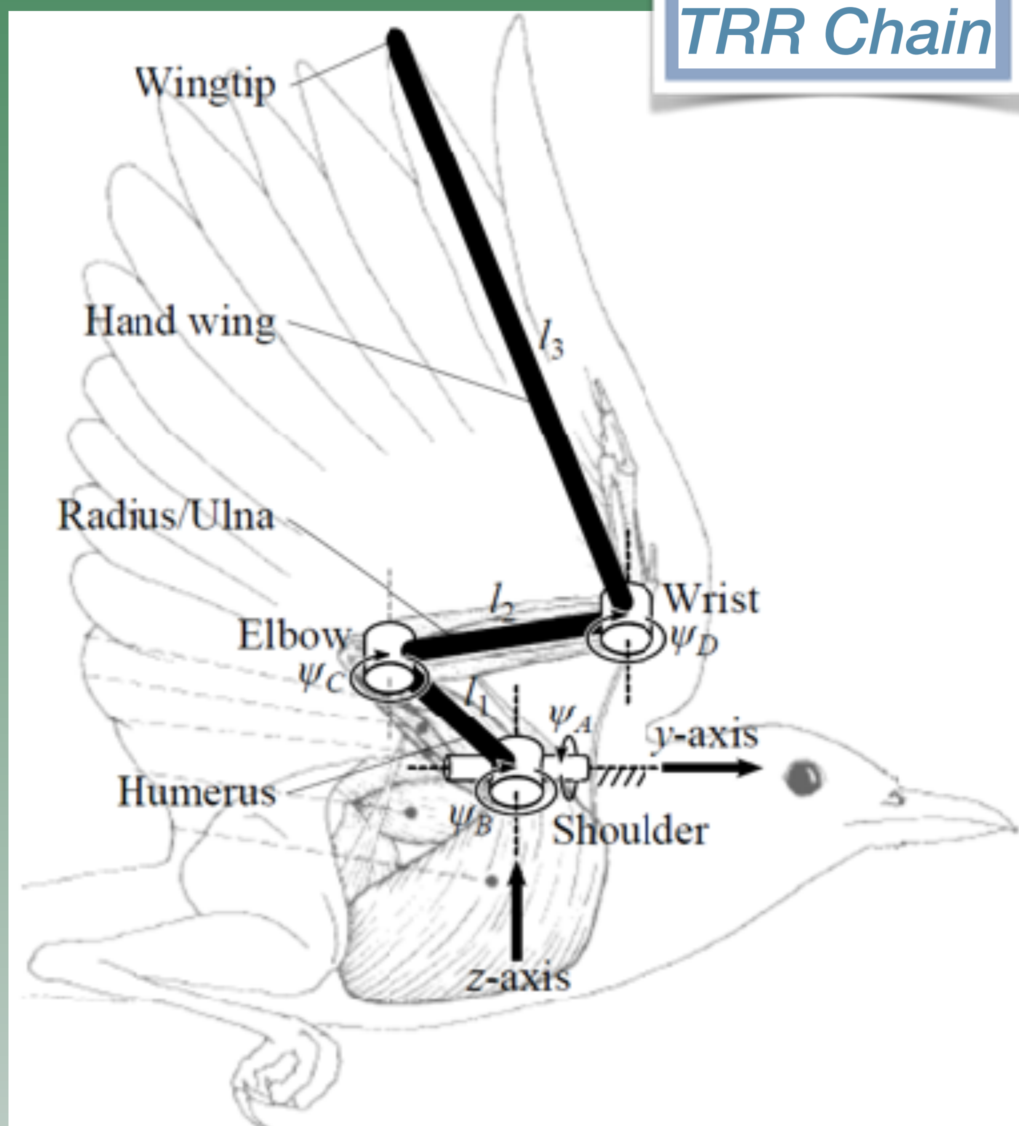
Data obtained from Tobalske and Dial high speed video footage of a black-billed magpie flying

	Wrist	Wingtip
	{X, Y, Z}	{X, Y, Z}
1	{1.79, 0.80, 3.55}	{5.58, -2.87, 9.37}
2	{3.11, -0.05, 1.61}	{8.97, -0.04, 6.52}
3	{3.37, -0.61, -0.24}	{10.77, 0.68, 2.26}
4	{2.98, -0.30, -1.18}	{9.63, 0.68, -1.10}
5	{2.10, 0.14, -1.40}	{5.49, -1.08, -6.33}
6	{1.48, 0.26, -0.46}	{1.66, -2.14, -7.49}
7	{0.91, 0.48, 1.08}	{1.35, -5.62, -3.92}
8	{0.57, 0.89, 2.24}	{2.67, -5.66, 3.42}

B. Tobalske, and K. Dial, 1996. "Flight kinematics of black-billed magpies and pigeons over a wide range of speeds," The Journal of Experimental Biology, 199(2):263–280.

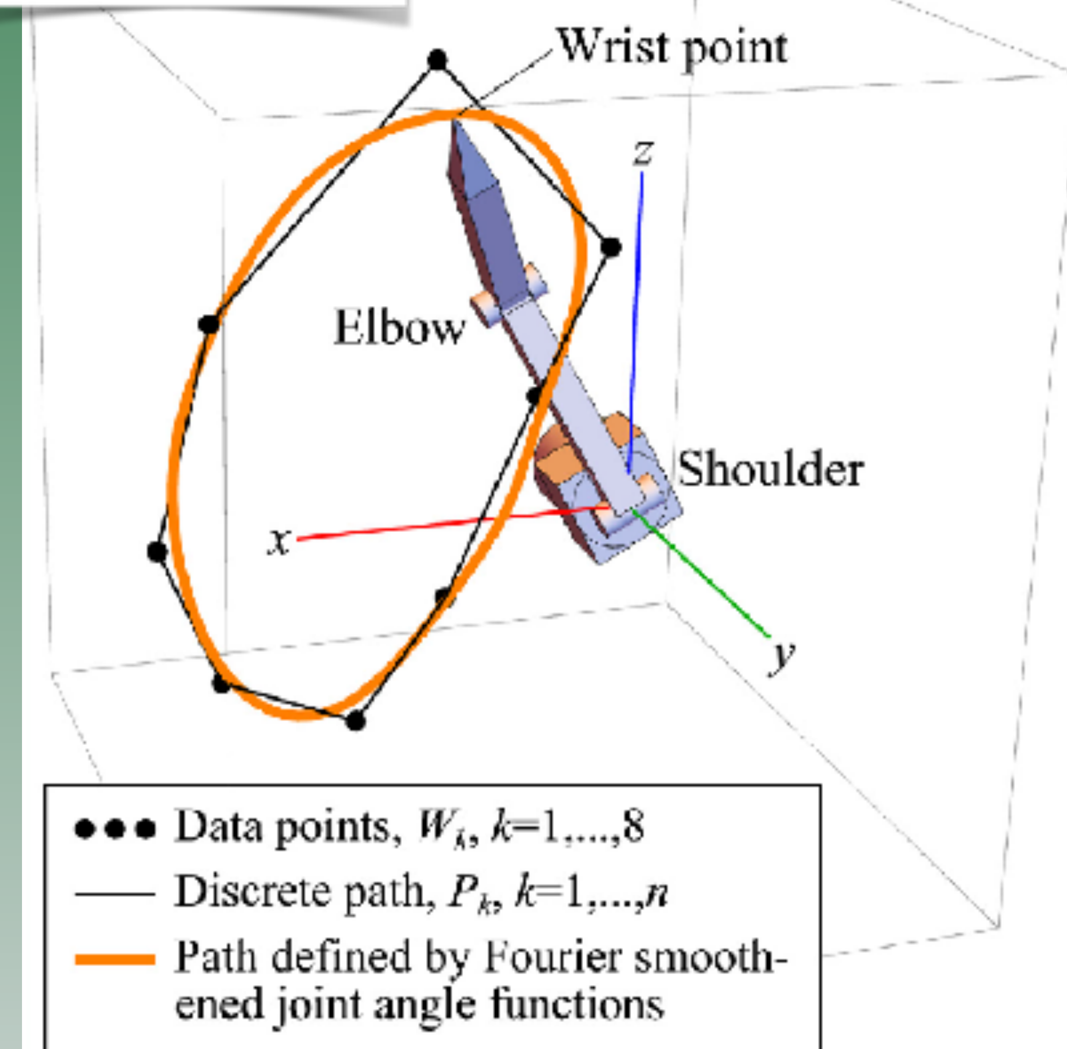
Mechanically Driven Spatial Chain

TRR Chain



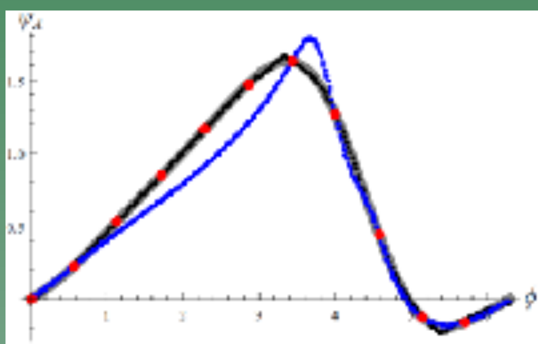
$$\hat{D}(\psi_A,\psi_B,\psi_C,\psi_D) = \hat{X}\left(-\frac{\pi}{2},0\right)\hat{Z}(\psi_A,0)\hat{X}\left(\frac{\pi}{2},0\right)\hat{Z}(\psi_B,0) \times \hat{X}(0,l_1)\hat{Z}(\psi_C-\psi_B,0)\hat{X}(0,l_2)\hat{Z}(\psi_D-\psi_C,0)\hat{X}(0,l_3)$$

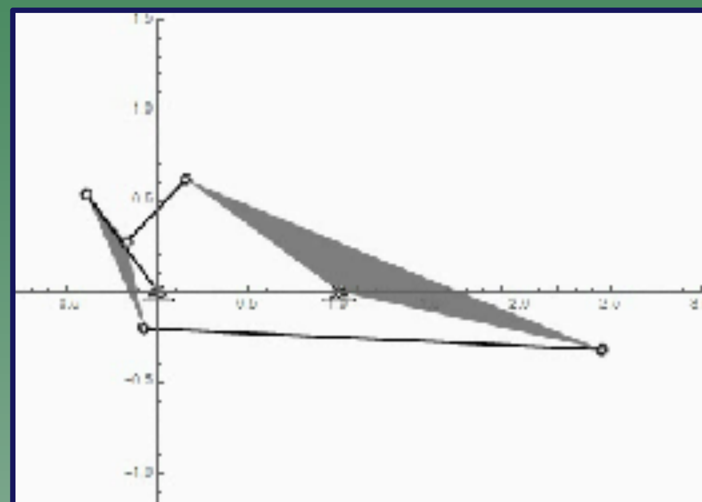
TR Subchain

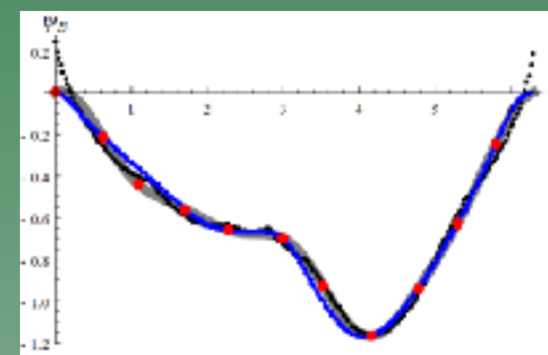


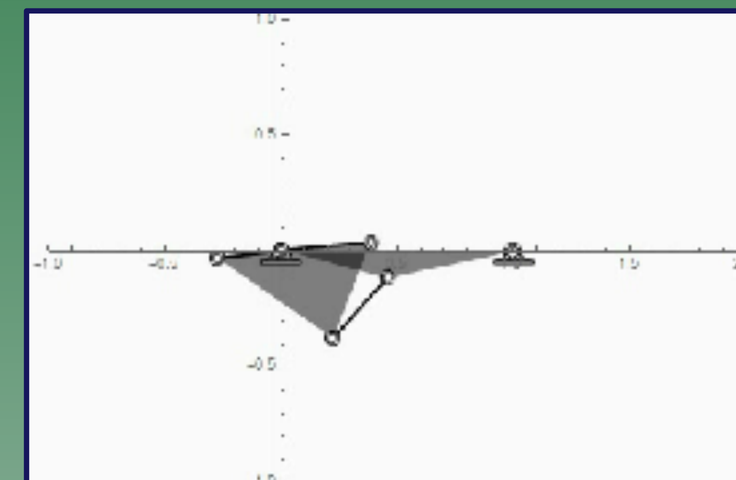
$$\hat{D}_{TR}(\psi_A,\psi_B,\psi_C) = \hat{X}(-\frac{\pi}{2},0)\hat{Z}(\psi_A,0)\hat{X}(\frac{\pi}{2},0)\hat{Z}(\psi_B,0) \times \hat{X}(0,l_1)\hat{Z}(\psi_C-\psi_B,0)\hat{X}(0,l_2)$$

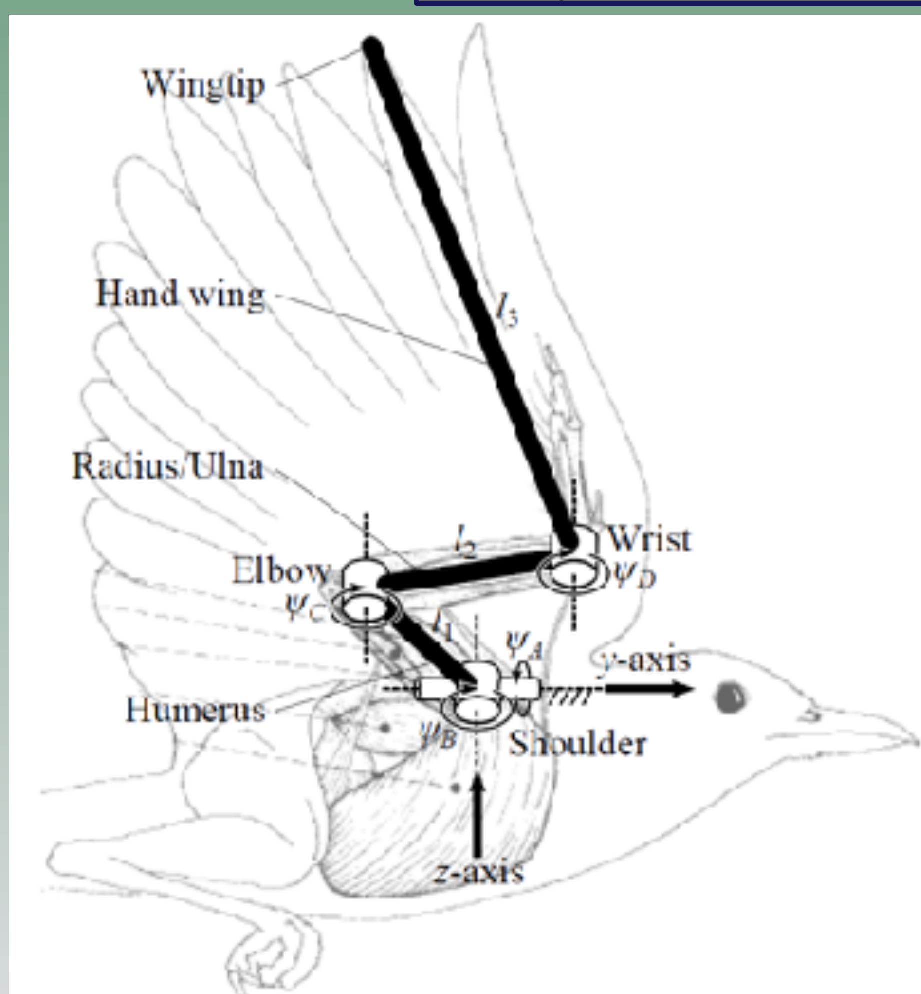
Driving Six-bar Linkages

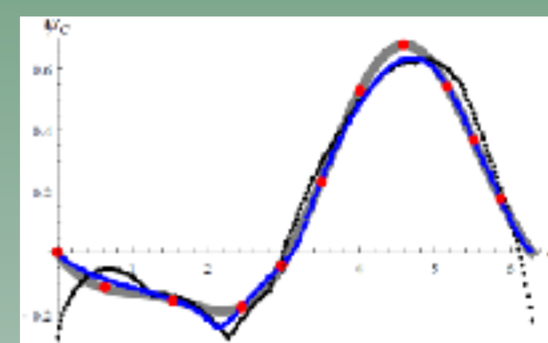

(a) Joint angle ψ_A

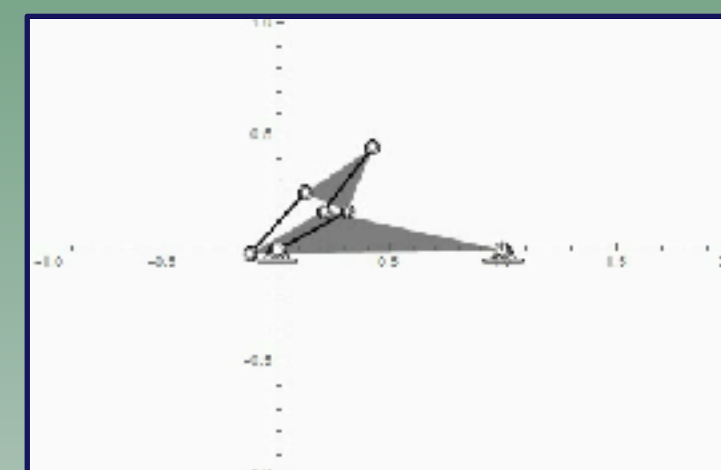


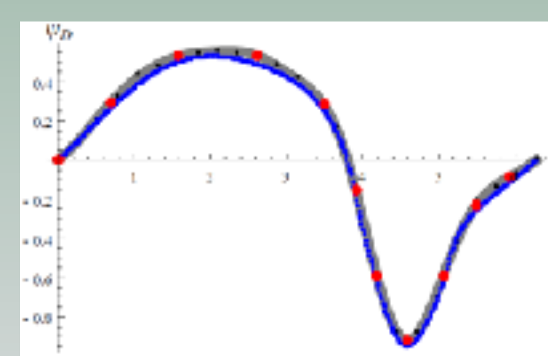

(b) Joint angle ψ_B

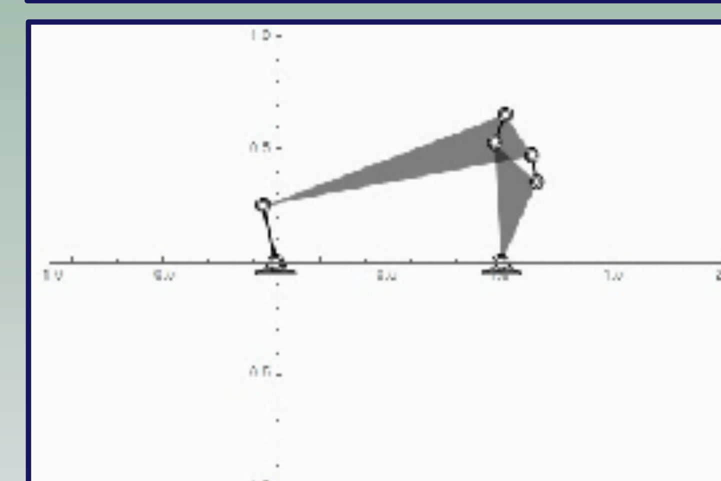





(c) Joint angle ψ_C



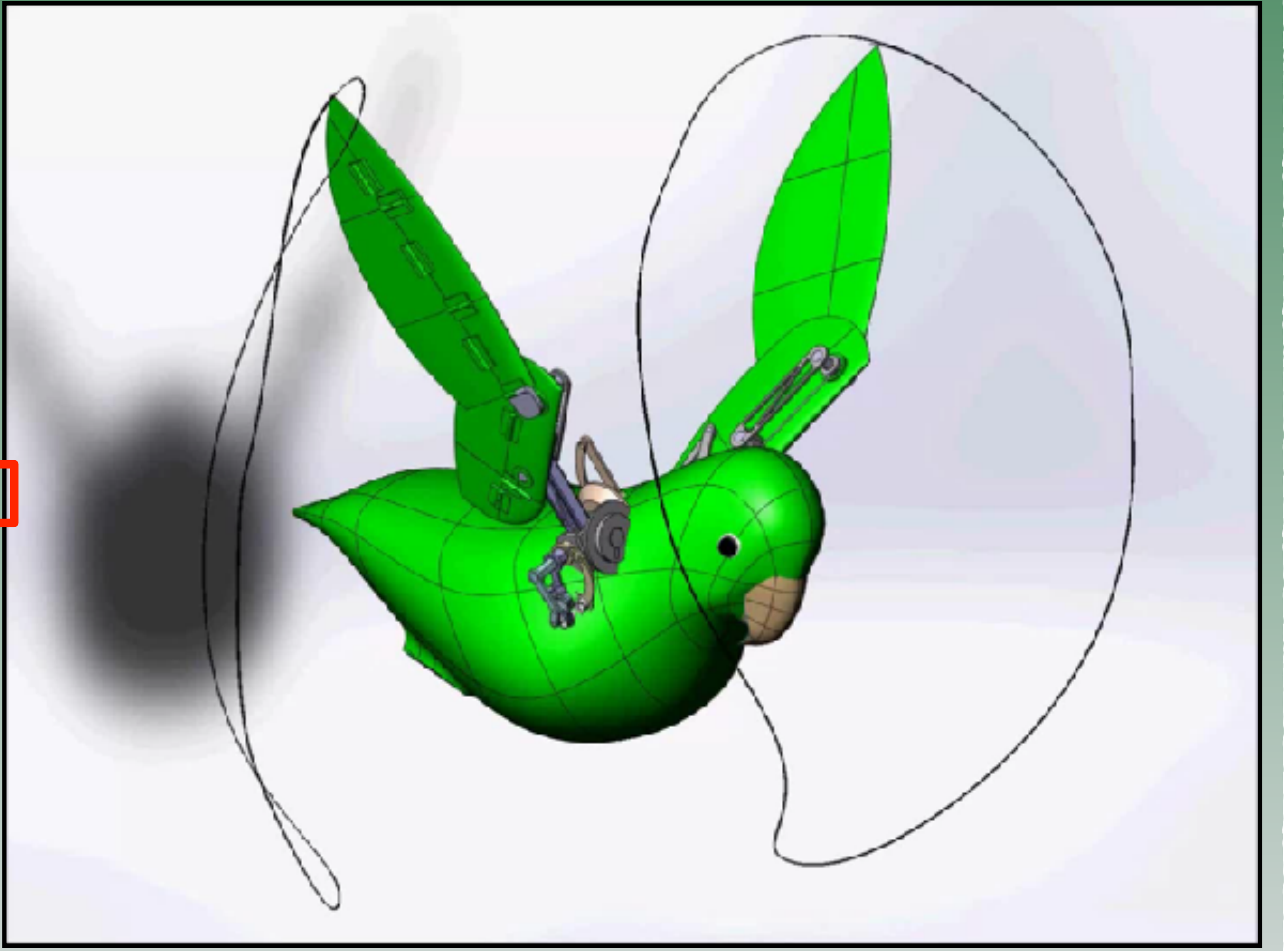

(d) Joint angle ψ_D



Biomimetic Movement

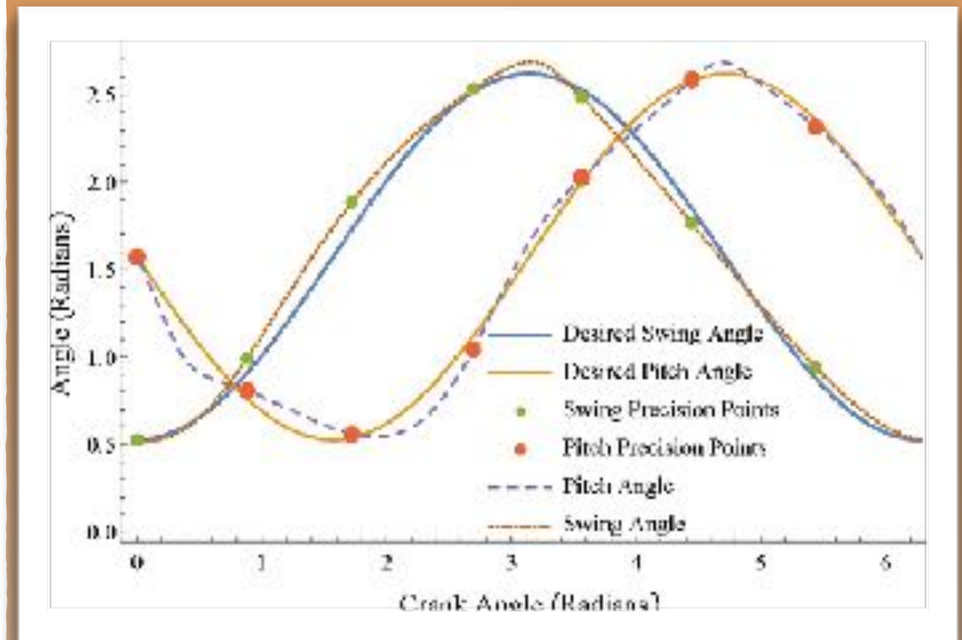
Solutions of the synthesis polynomials for each of the four target functions
$\psi_A=f_A(\phi)$, $\psi_B=f_B(\phi)$, $\psi_C=f_C(\phi)$, $\psi_D=f_D(\phi)$

	Binary Driven			
	ψ_A	ψ_B	ψ_C	ψ_D
Linkage solutions	11428	7215	12870	11693
Design Candidates	6068	4012	7363	5775
11 point mechanisms	0	0	3	0
10 point mechanisms	0	0	12	0
9 point mechanisms	0	7	95	4
8 point mechanisms	21	54	246	95
Feasible designs	21	61	356	99
Synthesis computation time (hr)	2.2	2.0	2.5	2.2
Analysis computation time (hr)	20.2	13.4	25.6	19.1

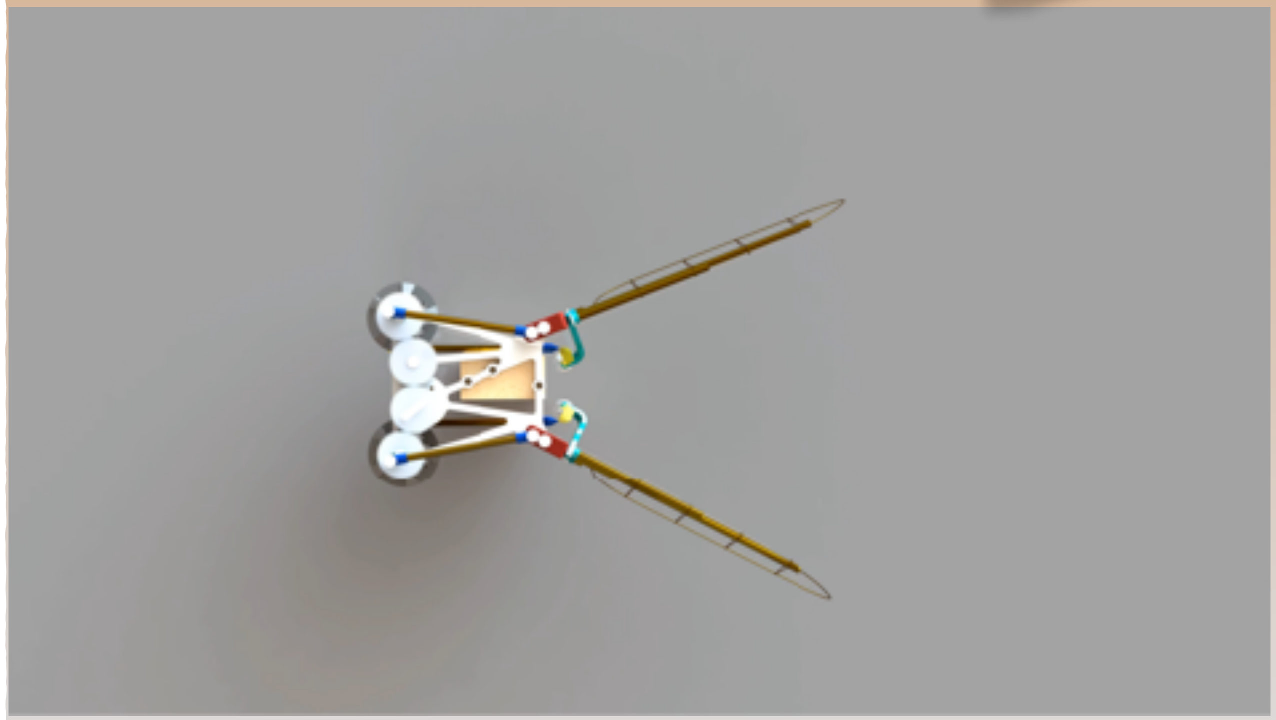


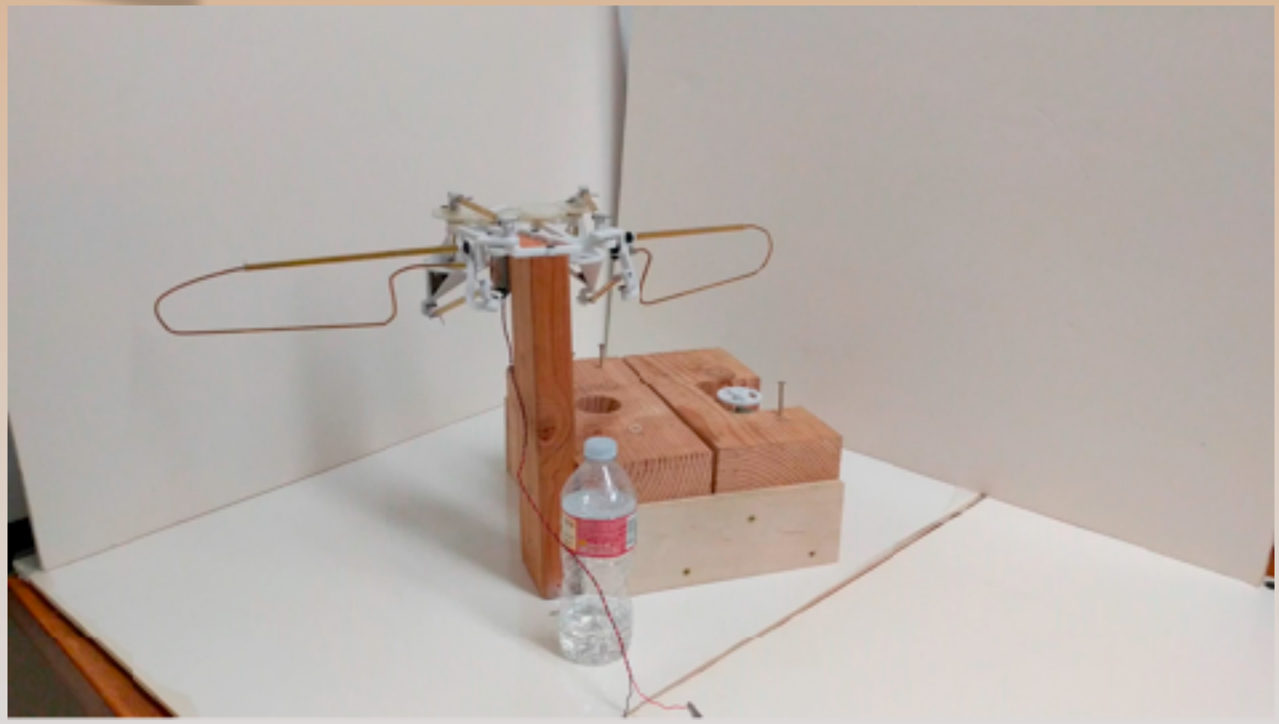
As many as 45×10^6 possible wing linkage designs.

Hovering Wing Movement



Balta,M.,Ahmed,K.A.,Wang,P.L.,McCarthy,J.M., and Taha, H. E., 2017. "Design and manufacturing of flapping wing mechanisms for micro air vehicles". In 58th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, p. 0509.





Spatial Mechanisms

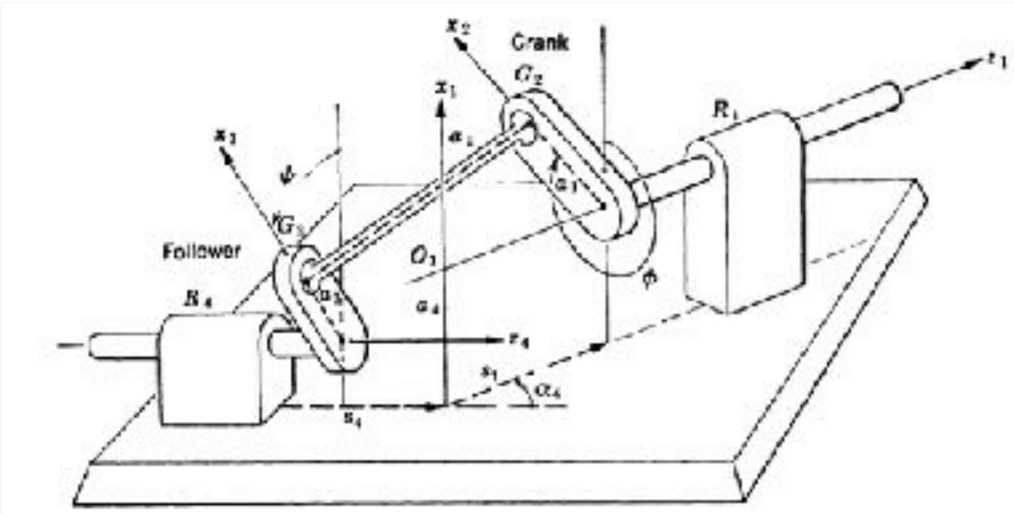
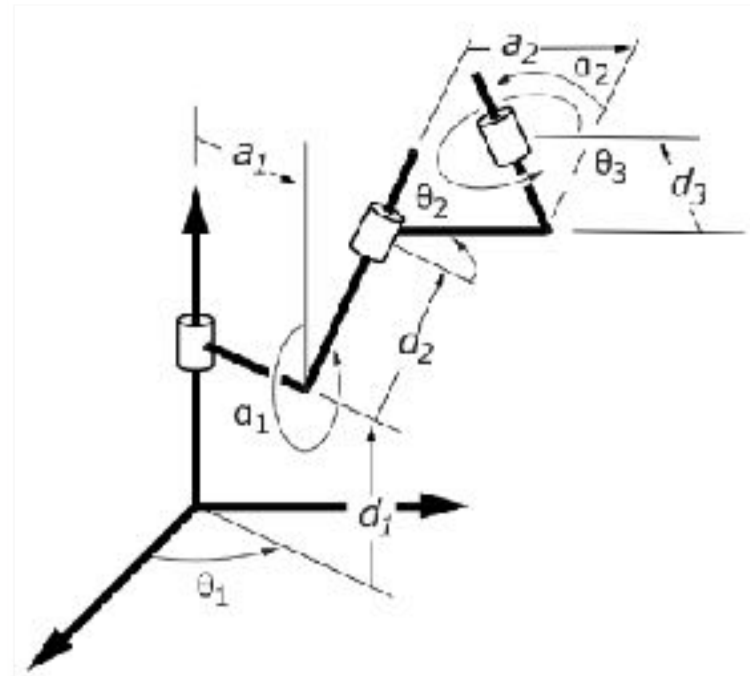
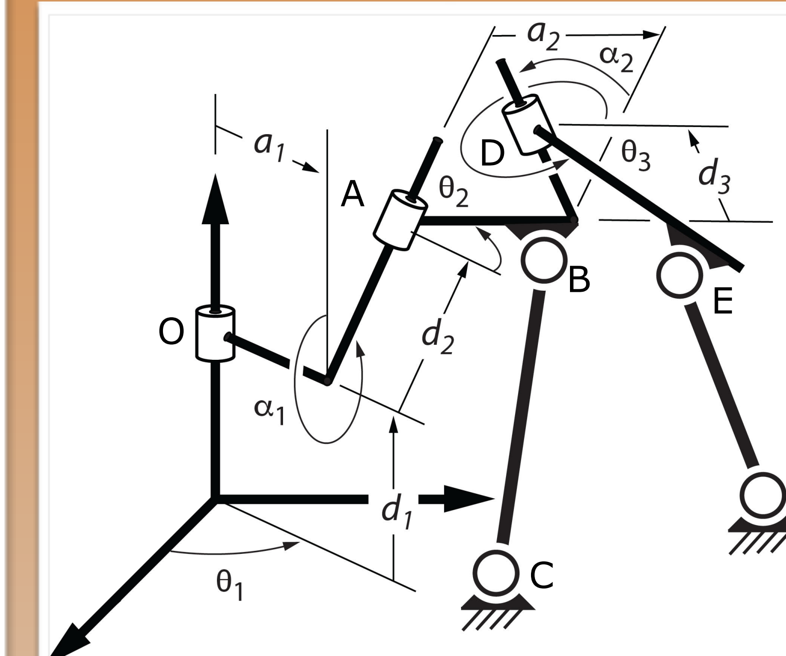
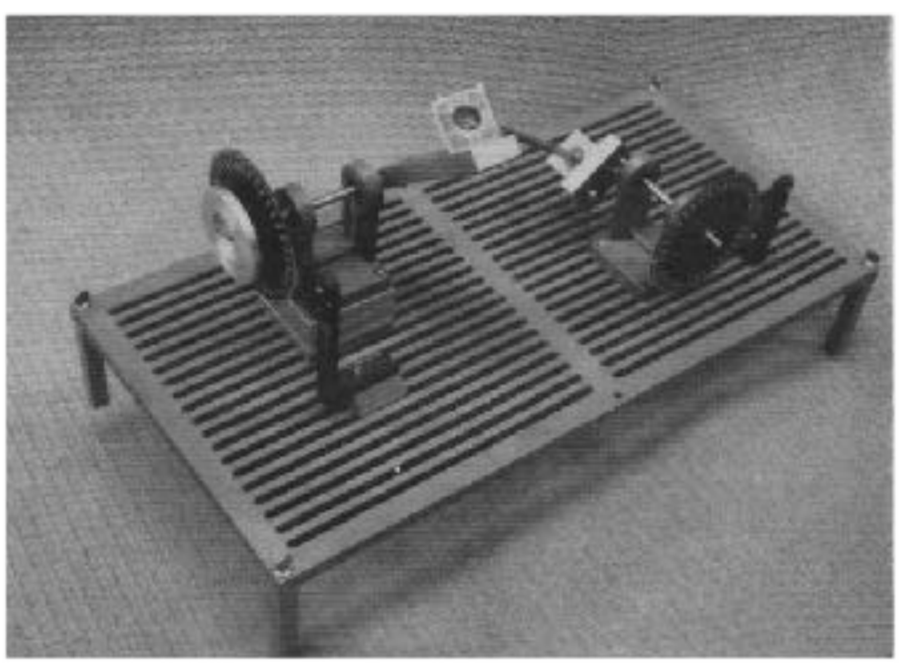
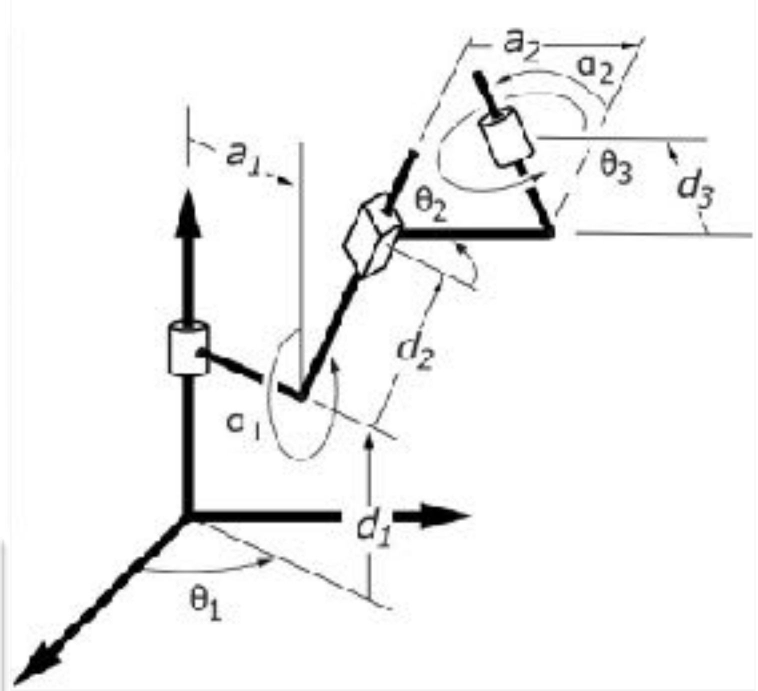


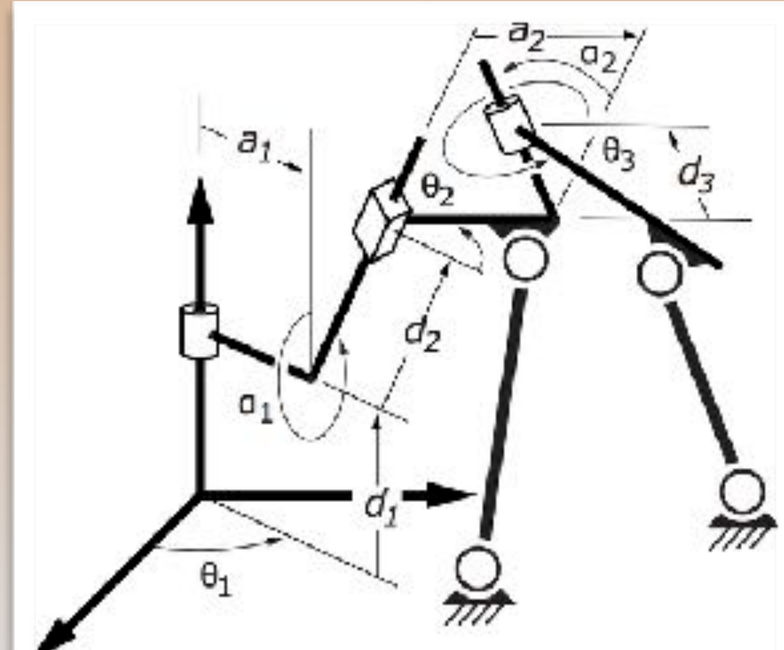
FIGURE 12-1 Two-revolute two-spheric-pair mechanism.





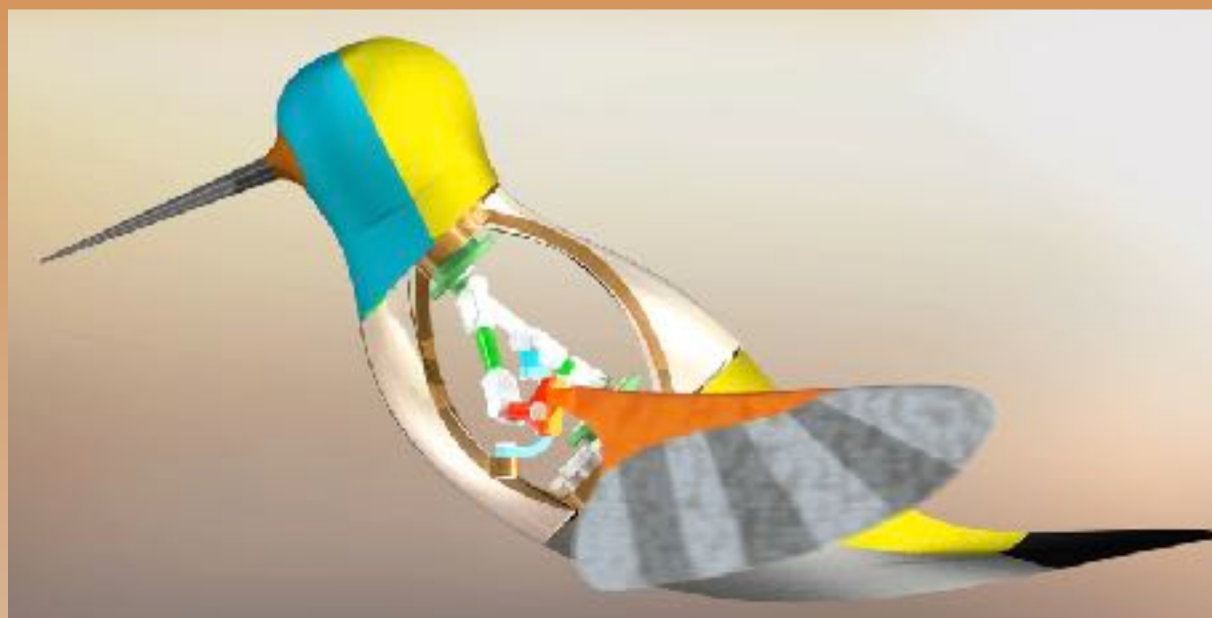


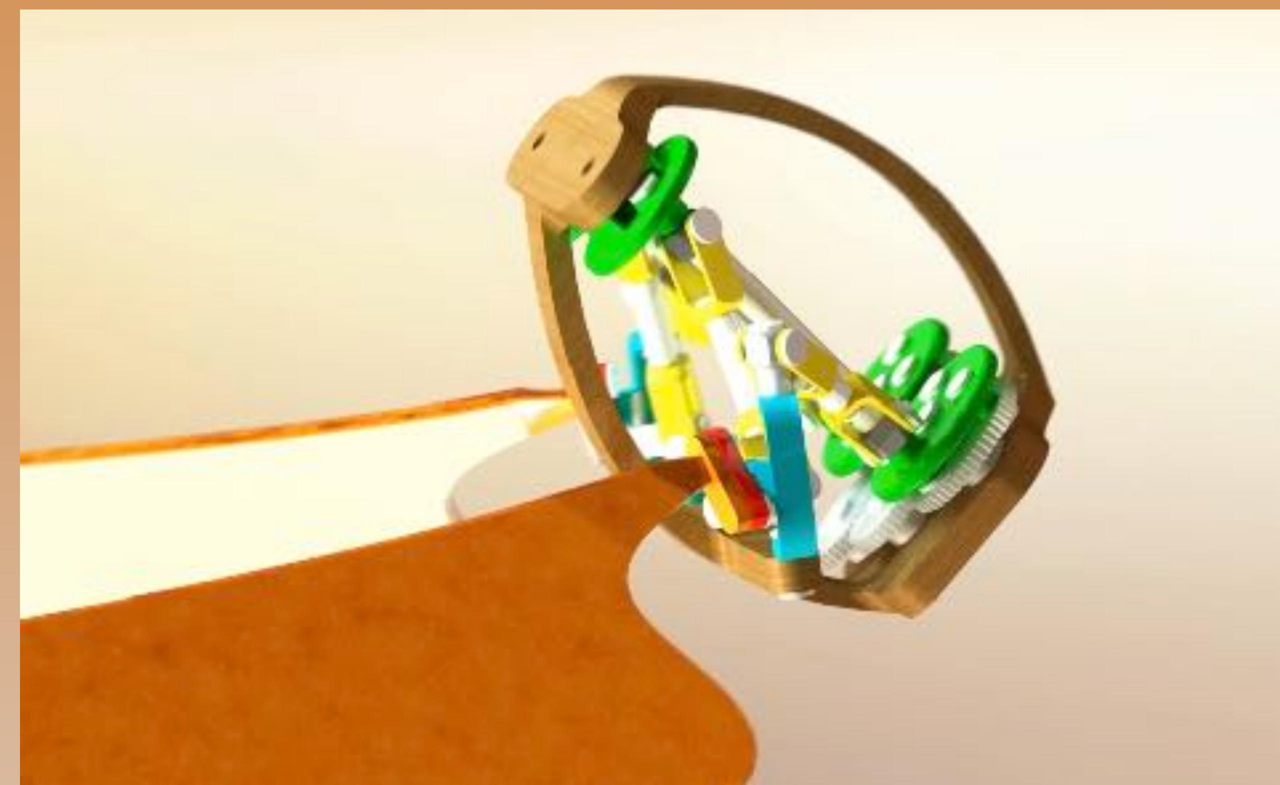


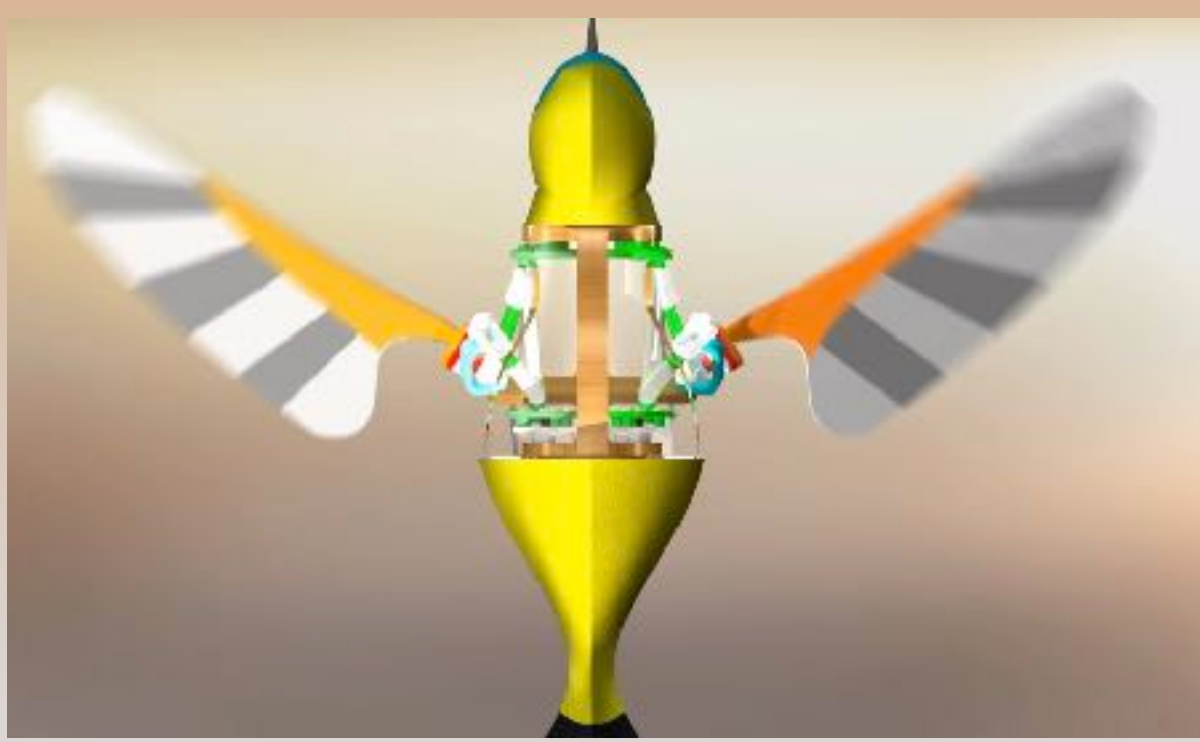


R. S. Hartenberg and J. Denavit, **1964** *Kinematic Synthesis of Linkages*, McGraw-Hill, New York.

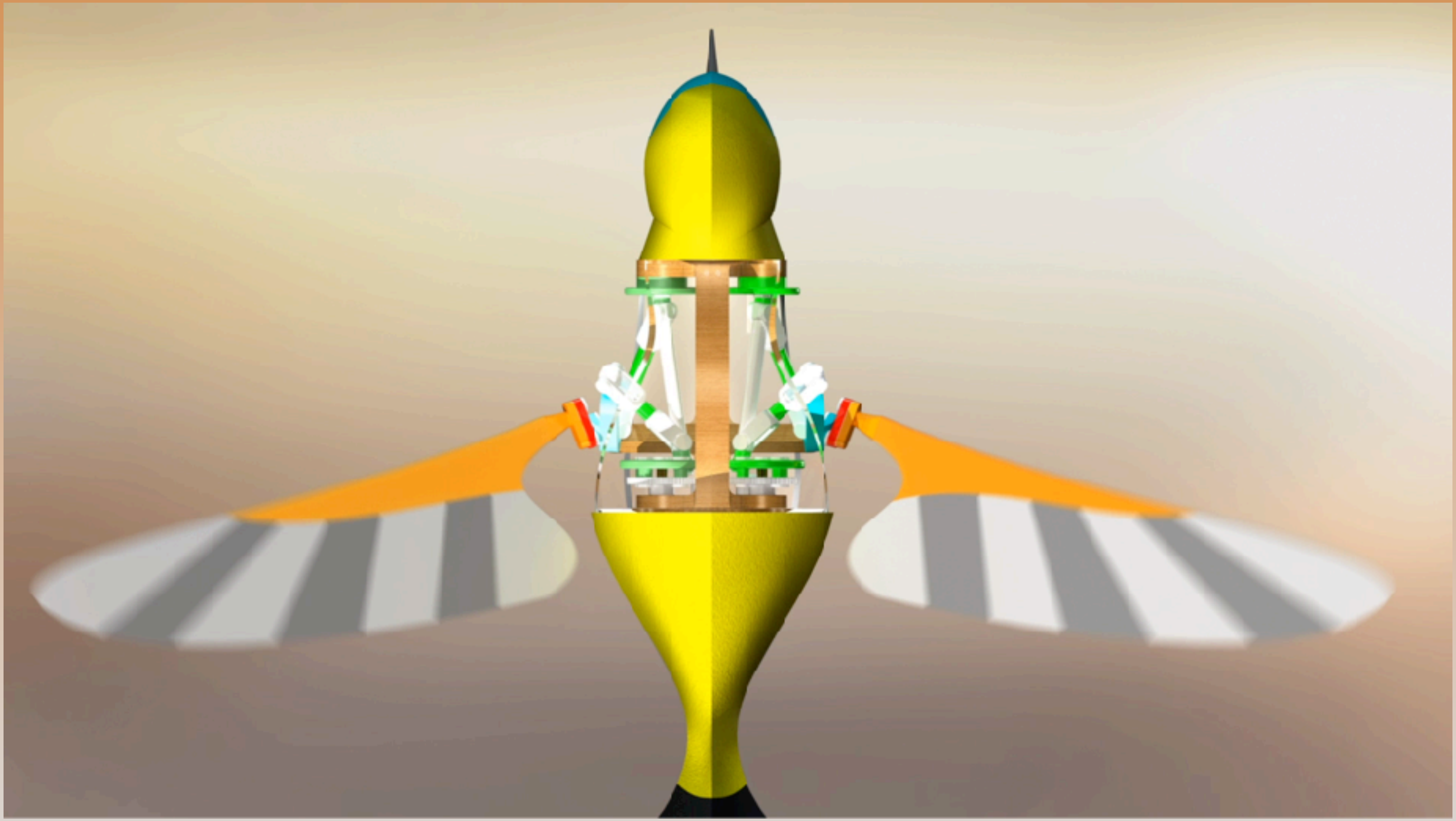
New Flapping Wing Mechanism



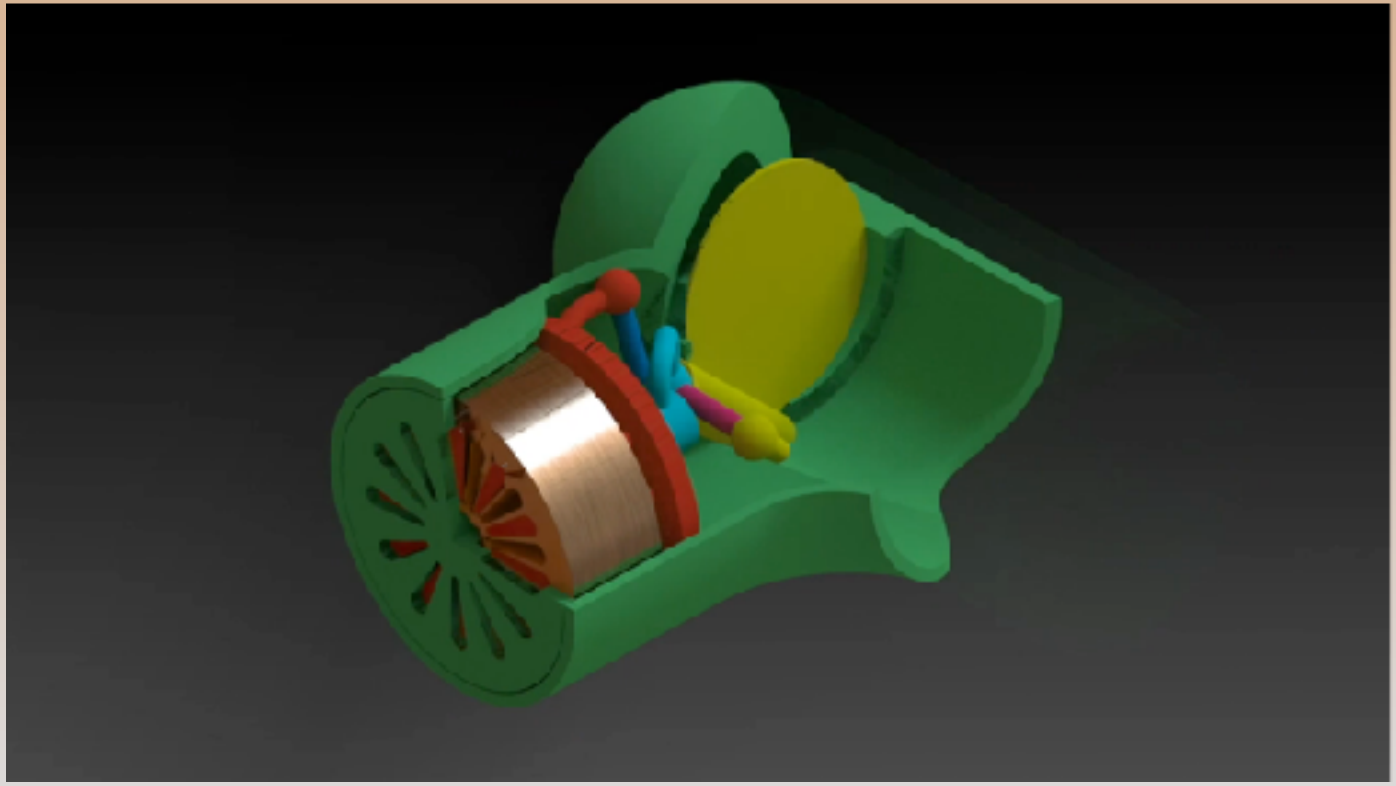




Flapping Wing Movement



Tunnel Boring Machine



Inventors


Vital Link
2017
UCI ENERGY INVITATIONAL
Ei8
DESIGN REVIEW
APRIL 21, 2017, 9AM - 1PM
OC FAIR & EVENT CENTER
PRACTICE RUN
APRIL 29, 2017, 8AM - 1PM
UCI LOT 16H
COMPETITION DAY
MAY 6, 2017, 8AM - 3PM
UCI LOT 16
HOW FAST CAN YOU GO ON
ONE DOLLAR'S WORTH OF ENERGY?


Ei7
THE UNIVERSITY
OF CALIFORNIA IRVINE

Conclusions

- *Kempe's Universality Theorem, proves the existence of a drawing linkage for every algebraic curve, but the construction yields complex systems with hundreds of links for a cubic curve.*
- *Trigonometric cubic Bezier curves can be drawn by four-link coupled serial chains. Thus, a one degree-of-freedom linkage system can sign your name and write cursive Chinese.*
- *Path synthesis of a four-bar coupler curve through nine points has been solved. Path synthesis of a six-bar coupler curve through 15 points is unsolved, because is beyond our computational capabilities.*
- *Kinematic synthesis of robotic systems to draw curves contributes to design innovation. Opportunities are as varied as stroke rehabilitation, disaster relief, vehicle suspensions, and walking and flying robots.*
- *Spatial Mechanism Synthesis is yielding innovative designs for new applications in areas as diverse as micro-air vehicles and tunnel boring machines.*

Thank you, do you have any questions?